VETRI VINAYAHA COLLEGE OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF MATHEMATICS
MA6451 PROBABILITY & RANDOM PROCESSES

BRANCH :II ECE **SEMESTER: IV**

PART - A QUESTION AND ANSWER
UNIT I - RANDOM VARIABLES

1. **Define random variable. Give an example.**

Solution: A random variable is a function which assigns a real number for every outcome of an experiment. **Ex**: In tossing two coins, let X denotes the number of heads.

Sample Space (S):	HH	HT	TH	TT
X:	2	1	1	0

2. **Write any two properties of CDF F(x).**

Solution: $i) 0 \le F(x) \le 1, \quad -\infty < x < \infty$ $ii) \phi(x_1 \le X < x_2) = F(x_2) - F(x_1)$

$iii) F(\infty) = \underset{x \to \infty}{lt} F(x) = 1$ $iv) F(-\infty) = \underset{x \to -\infty}{lt} F(x) = 0$.

3. **What are the two types of moments?**

Solution: i) Moments about the origin (Raw Moments) ii) Moments about the Mean (Central Moments)

4. **Give the mean and variance of Poisson distribution.**

Solution: Mean =Variance= λ.

5. **Define Binomial distribution.**

Solution: The random variable X is said to follow binomial distribution if it assumes only non-negative values and its probability mass function is $p(X = x) = nC_x p^x q^{n-x}$.

6. **Define Poisson distribution.**

Solution: The random variable X is said to follow Poisson distribution if it assumes only non-negative values and its probability mass function is $P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}$.

7. **Define Geometric distribution.**

Solution: The random variable X is said to follow Geometric distribution if it assumes only non-negative values and its probability mass function is given by $P(X = x) = pq^{x-1}, \quad x = 1, 2, ...$

8. **Define Negative Binomial distribution.**

Solution: The random variable X is said to follow Negative Binomial distribution if it assumes only non- negative values and its probability mass function is given by

$P(X = x) = x + r - 1C_{r-1} p^r q^x, \quad x = 0, 1, 2, ...$

9. **What is the pdf of an uniform distribution.**

Solution: A continuous random variable X is said to follow an uniform distribution with interval (a,b) if it's the probability density function is $f(x)=\begin{cases}\dfrac{1}{b-a}, & a<x<b\\ 0\ , otherwise\end{cases}$

10. **What is the pdf of an exponential distribution.**

Solution: A continuous random variable X is said to follow an exponential distribution with parameter λ>0 if it's the probability density function is given by $f(x)=\begin{cases}\lambda e^{-\lambda x}, & x\geq 0\\ 0\ , otherwise\end{cases}$

11. **Define Gamma (or) Erlang distribution.**

Solution: A continuous random variable X is said to follow Gamma distribution with two parameters $\lambda>0,\ k>0$ if the probability density function is given by $f(x)=\dfrac{\lambda^k\ x^{k-1}\ e^{-\lambda x}}{\overline{)k}}, k\geq 0$.

12. **Define Weibull distribution.**

Solution: A random variable X is said to follow Weibull distribution with two parameters $\alpha>0,\ \beta>0$ if the probability density function is given by $f(x)=\alpha\beta x^{\beta-1}e^{-\alpha x^{\beta}}\ , x>0$.

13. **What is the relationship between Weibull and Exponential distribution?**

Solution: The pdf of Weibull distribution is $f(x)=\alpha\beta x^{\beta-1}e^{-\alpha x^{\beta}}\ , x>0$ **------(1)**

When $\beta=1$ (1) becomes $f(x)=\alpha\ e^{-\alpha x}, x>0$ which is the pdf of exponential distribution with parameter α .

14. **If a random variable X takes the values 1,2,3,4 such that 2P(X=1)=3P(X=2)=P(X=3)=5P(X=4). Find the probability distribution of X.**

Solution: Let 2P(X=1)=3P(X=2)=P(X=3)=5P(X=4) = a.

$$\Rightarrow 2P(X=1)=a \Rightarrow P(X=1)=\frac{a}{2},\quad 3P(X=2)=a \Rightarrow P(X=2)=\frac{a}{3},\quad P(X=3)=a,\quad 5P(X=4)=a \Rightarrow P(X=4)=\frac{a}{5}$$

x:	1	2	3	4
$P[X=x]$:	$\frac{a}{2}$	$\frac{a}{3}$	a	$\frac{a}{5}$

To find a: Wkt $\sum p(x)=1$ *ie)* $\frac{a}{2}+\frac{a}{3}+a+\frac{a}{5}=1 \Rightarrow \frac{15a+10a+30a+6a}{30}=1 \Rightarrow a=\frac{30}{61}$.

Probability distribution of X:

x:	1	2	3	4
$P[X=x]$:	$\frac{15}{61}$	$\frac{10}{61}$	$\frac{30}{61}$	$\frac{6}{61}$

15. **If X is a discrete random variable with probability distribution P[X=x] =kx, x=1,2,3,4. Find P(2<X<4).**

Solution: To find k: Wkt $\sum p(x)=1$ *ie)* $k+2k+3k+4k=1 \Rightarrow 10k=1 \Rightarrow k=\frac{1}{10}$

To find P(2<X<4): P(2<X<4)=P(X=3)=3/10.

16. **If the range of X is the set $\{0,1,2,3,4\}$ and P[X=x]=0.2 for $x\in X$ determine the mean and variance of the random variable.(2)**

Solution: $Mean=E[X]=\sum x\,p(x)=0.2[0+1+2+3+4]=2$

$$E[X^2] = \sum x^2 \; p(x) = 0.2[0^2 + 1^2 + 2^2 + 3^2 + 4^2] = 6$$
$$Var[X] = E[X^2] - \left(E[X]\right)^2 = 6 - 4 = 2$$

17. **If a random variable X has the distribution function** $F(X) = \begin{cases} 1 - e^{-\alpha x}, & for \;\; x > 0 \\ 0, & for \;\; x \le 0 \end{cases}$ **where** α **is a parameter. Find** $P(1 \le X \le 2).$

Solution: $P(1 \le X \le 2). = F(2) - F(1)$ $\quad\quad [\because F(X) = 1 - e^{-\alpha x}]$
$$= (1 - e^{-2\alpha}) - (1 - e^{-\alpha}) = e^{-\alpha} - e^{-2\alpha}$$

18. **A random variable X has the p.d.f f(x) given by** $f(x) = \begin{cases} C\,x\,e^{-x}, & x > 0 \\ 0, \; if & x \le 0 \end{cases}$**. Find the value of C.**

Solution: Given $f(x) = C\,x\,e^{-x}, \; x > 0$

To find C: WKT $\int_{-\infty}^{\infty} f(x)dx = 1 \Rightarrow \int_{0}^{\infty} C\,x\,e^{-x}dx = 1 \Rightarrow C\overline{)2} = 1 \Rightarrow C = 1$ $\quad\quad [\because \overline{)2} = 1]$

19. **Let X be a random variable with p.d.f** $f(x) = \frac{1}{2} e^{-x/2}$**, X>0. Find P(x>3).**

Solution: Given $f(x) = \frac{1}{2} e^{-x/2}$

To Find P(x>3): $P(X > 3) = \int_{3}^{\infty} f(x)\,dx \;\; = \int_{3}^{\infty} \frac{1}{2} e^{-x/2}\,dx = \frac{1}{2}\left[\frac{e^{-x/2}}{-\frac{1}{2}}\right]_3^{\infty} = -e^{-\infty/2} + e^{-3/2} = e^{-3/2}$

20. **A continuous random variable X has a probability density function** $f(x) = k(1+x),$ $2 < X < 5.$ **Find P(X<4). (or) A continuous random variable X that can assume any value between x=2 and x=5 has a density function given by** $f(x) = k(1+x),$ **Find P(X<4). (4)**

Solution: WKT $\int_{-\infty}^{\infty} f(x)dx = 1 \Rightarrow \int_{2}^{5} k(1+x)dx = 1$
$$k\left(x + \frac{x^2}{2}\right)_2^5 = 1 \Rightarrow k\left(\left(5 + \frac{25}{2}\right) - \left(2+2\right)\right) = 1$$
$$k\left(\frac{27}{2}\right) = 1. \Rightarrow k = \frac{2}{27}$$

To Find P(X<4): ie) $P(2 < X < 4) = \int_{2}^{4} k(1+x)dx = \frac{2}{27}\left[\left(x + \frac{x^2}{2}\right)_2^4\right] = \frac{2}{27}\left[\left(4+8\right) - (2+2)\right] = \frac{16}{27}.$

21. **A continuous random variable X has the p.d.f f(x) given by** $f(x) = C\; e^{-|x|}, -\infty < x < \infty$**. Find the value of C and C.D.F of X.**

Solution: Given $f(x) = C\; e^{-|x|}, -\infty < x < \infty$

To find C: WKT $\int_{-\infty}^{\infty} f(x)dx = 1 \Rightarrow \int_{-\infty}^{0} C\,e^{x}dx + \int_{0}^{\infty} C\,e^{-x}dx = 1 \Rightarrow C = \frac{1}{2}$

$$\therefore\ f(x)=\begin{cases}\frac{1}{2}e^{x} & if\ \ x\ge 0\\ \frac{1}{2}e^{-x} & if\ \ x<0\end{cases}$$

C.D.F of X F(x) = $\int_{-\infty}^{x} f(x)dx$

$$if\ \ x\le 0 \qquad F(x)=\int_{-\infty}^{x}\frac{1}{2}e^{x}dx=\frac{e^{x}}{2}$$

$$if\ \ x>0 \qquad F(x)=\int_{-\infty}^{0}\frac{1}{2}e^{x}dx+\int_{0}^{x}\frac{1}{2}e^{-x}dx=1-\frac{e^{-x}}{2}$$

$$\therefore\ \ C.D.F\ \ of\ \ F(x)=\begin{cases}\frac{e^{x}}{2} & if\ \ x\le 0\\ 1-\frac{e^{-x}}{2} & if\ \ x>0\end{cases}$$

22. **What is the relationship between raw moments and central moments.**

Solution: i) $\mu_1=0$ (Always) ii) $\mu_2=\mu_2'-\left(\mu_1'\right)^2$

$iii)\ \mu_3=\mu_3'-3\mu_2'\ \mu_1'+2\left(\mu_1'\right)^3$ $\qquad iv)\ \mu_4=\mu_4'-4\mu_3'\ \mu_1'+6\mu_2'\left(\mu_1'\right)^2-3\left(\mu_1'\right)^4$

23. **The first four moments of a distribution about X=4 are 1,4,10 and 45 respectively. Show that mean is 5, variance is 3, $\mu_3=0$ *and* $\mu_4=26$.**

Solution: Given $\mu_1'=1,\ \mu_2'=4,\mu_3'=10\ \ and\ \ \mu_4'=45$. The Point is X=4.

Mean =X+ $\mu_1'=4+1=5$, Variance = $\mu_2=\mu_2'-\left(\mu_1'\right)^2=4-1=3$

$$\mu_3=\mu_3'-3\mu_2'\ \mu_1'+2\left(\mu_1'\right)^3=10-3(4)(1)+2(1)^3=0$$

$$\mu_4=\mu_4'-4\mu_3'\ \mu_1'+6\mu_2'\left(\mu_1'\right)^2-3\left(\mu_1'\right)^4=45-4(10)(1)+6(4)(1)^2-3(1)^4=26.$$

24. **If a random variable X has the MGF $M_X(t)=\frac{3}{3-t}$. Find the standard deviation of X.**

Solution: Given $M_X(t)=\frac{3}{3-t}\Rightarrow M_X(t)=3\left(3-t\right)^{-1}$

$$Mean=E[X]=\frac{d}{dt}\left[M_X(t)\right]_{t=0}=\frac{d}{dt}\left[3(3-t)^{-1}\right]_{t=0}=\left[3(3-t)^{-2}\right]_{t=0}=\frac{1}{3}$$

$$E[X^2]=\frac{d^2}{dt^2}\left[M_X(t)\right]_{t=0}=\frac{d}{dt}\left[3(3-t)^{-2}\right]_{t=0}=\left[6(3-t)^{-3}\right]_{t=0}=\frac{2}{9}$$

Variance = $E[X^2]-\left[E(X)\right]^2=\frac{1}{9}$.

Standard Deviation $=\sqrt{Var(X)}=\frac{1}{3}$.

25. **Find the mean and variance of the distribution whose moment generating function is $(0.4e^{t}+0.6)^{8}$.**

Solution: Given $M_X(t)=(0.4e^{t}+0.6)^{8}$ which is the MGF of binomial distribution $M_X(t)=(pe^{t}+q)^{n}$.

Here p=0.4, q=0.6, n=8. Mean = np=8 x 0.4 =3.2, Variance=npq=8 x 0.4 x 0.6=1.92.

26. **The moment generating function of a random variable X is given by $M_X(t)=e^{3(e^{t}-1)}$. Find P(X=1).**

Solution: Given $M_X(t)=e^{3(e^{t}-1)}$ which is the MGF of Poisson distribution $M_X(t)=e^{\lambda(e^{t}-1)}$

Here $\lambda = 3$. The Poisson distribution is $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$

$\therefore$ P[X=1]= $P(X = 1) = \frac{e^{-3}3^1}{1!} = 0.1494.$

27. **Find the binomial distribution for which mean 4 and variance 4/3. Find $P(X \geq 1)$.**

Solution: Given Mean=np=4------(1) Variance=npq=4/3.---(2)

$$\frac{(1)}{(2)} \Rightarrow q = \frac{1}{3}. \quad wkt \quad p + q = 1 \Rightarrow p = \frac{2}{3}$$

$$Subustitute \ \ p \ \ in \ \ (1) \Rightarrow n = 6.$$

Binomial distribution is $p(X = x) = nC_x p^x q^{n-x}$

To Find $P(X \geq 1)$: $P(X \geq 1) = 1 - P(X=0) = 1 - nC_0 p^0 q^{6-0} = 1 - q^6 = 1 - \left(\frac{1}{3}\right)^6 = 0.998$

28. **If X is a Poisson variate such that P(X=2)=9P(X=4)+90P(X=6) find the variance.(2)**

Solution: WKT $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$

Given P(X=2)=9P(X=4)+90P(X=6)

$$\frac{e^{-\lambda}\lambda^2}{2!} = 9\frac{e^{-\lambda}\lambda^4}{4!} + 90\frac{e^{-\lambda}\lambda^6}{6!}$$

$$\frac{1}{2} = 9\frac{\lambda^2}{4!} + 90\frac{\lambda^6}{6!} \Rightarrow \frac{1}{2} = 3\frac{\lambda^2}{8} + \frac{\lambda^6}{8}$$

$$\Rightarrow \lambda^6 + 3\lambda^2 - 4 = 0 \Rightarrow \lambda^2 = \frac{-3 \pm \sqrt{9+16}}{2} = 1 \ \ or \ \ -4$$

$$\therefore \lambda = \pm 1 \quad or \quad \lambda = \pm 2i.$$

Mean=Variance= λ=1. [Since λ is always positive and cannot be imaginary].

29. **If X is a Poisson RV such that P(X=1)=0.3 and P(X=2)=0.2. Find P(X=0).**

Solution: Poisson distribution is $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$

Given P[X=1]=0.3 $\Rightarrow P(X = 1) = \frac{e^{-\lambda}\lambda^1}{1!} = e^{-\lambda}\lambda = 0.3$ ----------(1)

P[X=2]=0.2 $\Rightarrow P(X = 2) = \frac{e^{-\lambda}\lambda^2}{2!} = e^{-\lambda}\lambda^2 = 0.2$ --------(2) $\frac{(2)}{(1)} \Rightarrow \lambda = \frac{2}{3}$

$$\therefore P(X = 0) = \frac{e^{-\left(\frac{2}{3}\right)}\left(\frac{2}{3}\right)^0}{0!} = e^{-\left(\frac{2}{3}\right)}$$

30. **Every week the average number of wrong number phone calls received by a certain mail order house is seven. What is the probability that they will receive two wrong calls tomorrow?**

Solution: Let X be the Poisson random variable with $\lambda = \frac{7}{7} = 1$.

WKT $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$ $\therefore P(X = 2) = \frac{e^{-1}(1)^2}{2!} = \frac{e^{-1}}{2} = 0.18$

31. **The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = \frac{1}{2}$. What is the probability that a repair takes atleast 10 hours given that its duration exceeds 9 hours.**

Solution: Given Mean , $\lambda = \frac{1}{2}$**.** Exponential Distribution is $f(x) = \lambda e^{-\lambda x}, \quad x \ge 0$

$P(X > 10 / X > 9) = P(X > 1)$ **[** by Memory less property $P(X > n + m / X > n) = P(X > m)$ **]**

$$= \int_1^\infty f(x)\,dx = \int_1^\infty \frac{1}{2} e^{-\frac{x}{2}} dx = \frac{1}{2}\left[\frac{e^{-\frac{x}{2}}}{-\frac{1}{2}}\right]_1^\infty = e^{-\frac{1}{2}} = 0.6065$$

32. **A fast food chain finds that the average time customers have to wait for service is 45 seconds. If the waiting time can be treated as an exponential random variable, what is the probability that a customer will have to wait more than 5 minutes given that already he waited he waited for 2 minutes?**

Solution: Given Mean , $\lambda = \frac{1}{45}$ Exponential Distribution is $f(x) = \lambda e^{-\lambda x}, \quad x \ge 0$

$P(X > 5 / X > 2) = P(X > 3)$ **[** by Memory less property $P(X > n + m / X > n) = P(X > m)$ **]**

$$= \int_3^\infty f(x)\,dx = \int_3^\infty \frac{1}{45} e^{-\frac{x}{45}} dx = \frac{1}{45}\left[\frac{e^{-\frac{x}{45}}}{-\frac{1}{45}}\right]_3^\infty = e^{-\frac{3}{45}} = 0.9355$$

33. **If on the average rain falls on 10 days in every 30 days, obtain the probability that rain will fall on atleast 3 days of a given week.**

Solution: Given p=10/30=1/3, q=2/3, n=7.

WKT Binomial distribution is $p(X = x) = nC_x p^x q^{n-x}$

To find P[There is rainfall on at least 3 days]:

$ie)\ P[X \ge 3] = 1 - P[X < 3] = 1 - [P[X = 0) + P[X = 1) + P[X = 2]]$

$$= 1 - \left[\left(\frac{2}{3}\right)^7 + 7\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^6 + 21\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^5\right] = 1 - 0.5705 = 0.4295.$$

34. **The probability of a successful optical alignment in the assembly of an optical storage product is 0.8. Assume the trials are independent, what is the probability that the first successful alignment requires exactly four trials?**

Solution: Given p=0.8, q=0.2

By Geometric distribution $P[X=x]=pq^{x-1}$.

To Find P[Exactly four trials]:

$P[X=4]=(0.8)(0.2)^{4-1}=0.0064$.

35. **Suppose X has an exponential distribution with mean equal to 10. Determine the value of x such that P[X<x]=0.95.**

Solution: Given mean $\frac{1}{\lambda}=10 \Rightarrow \lambda=\frac{1}{10}$ Exponential Distribution is $f(x)=\lambda e^{-\lambda x}, \quad x \geq 0$

Given $\lambda e^{-\lambda x}=0.95 \Rightarrow \frac{1}{10} e^{-\frac{1}{10}x}=0.95$

$e^{-\frac{1}{10}x}=9.5 \Rightarrow -\frac{1}{10}x=\log 9.5 \Rightarrow -\frac{1}{10}x=0.9977 \Rightarrow x=-9.777$

36.If the probability that a target is destroyed on any one shot is 0.5 what is the probability that it would be destroyed on 6^{th} attempt?

Solution:p=0.5; q=1-p=1-0.5=0.5

$P(X=r)=q^{r-1}p$

$P(X=6)= q^5p=(0.5)^5(0.5)=0.01563$

UNIT II - TWO DIMENSIONAL RANDOM VARIABLES

1. **Define independence of two random variables X and Y, both in the discrete case and in the continuous case.**

Solution: Let X and Y are two independent random variables then in

i)Discrete Case: $P_{ij}=(P_{i.})(P_{.j})$ **ii)Continuous Case:** $f(x,y)=f(x)\ f(y)$

2. **List any two properties of joint distribution.**

Solution: Let X and Y are two independent random variables then

$i) F_{X,Y}(\infty,\infty)=1 \qquad ii) F_{X,Y}(-\infty,-\infty)=0$

$iii) 0 \leq F_{X,Y}(x,y) \leq 1 \quad iv) F_{X,Y}(x,\infty)=F_X(x),\ F_{X,Y}(\infty,y)=F_Y(y).$

3. **Define Marginal Probability Density function. (or)How do you find marginal density function of X and Y from the marginal probability functions $f_{XY}(x,y)$.**

Solution: The Marginal density function of X is $f(x)=f_X(x)=\int_{-\infty}^{\infty} f_{XY}(x,y)\ dy$

The Marginal density function of Y is $f(y)=f_Y(y)=\int_{-\infty}^{\infty} f_{XY}(x,y)\ dx$

4. **Define covariance (or) Let (X,Y) be a two dimensional random variable. Define covariance of (X,Y). If X and Y are independent, what will be the covariance of (X,Y)?**

Solution: If X and Y are the two random variables then the covariance between the two random variables are cov (X,Y)=E(XY)-E(X)E(Y).

If X and Y are independent random variables then cov (X,Y)=0.

5. **Comment on the following: "The random variables X and Y are independent iff Cov(X,Y)=0".**

Solution: If X and Y are random variables, then Cov (X,Y)=E(XY)-E(X)E(Y)

If X and Y are independent random variables then E(XY)=E(X)E(Y) **.**

$\therefore$ Cov (X,Y)=0. But the converse is not true.

6. **Define correlation between the random variables X and Yand write the types of Correlation.**

Solution: If the quantities (X,Y) vary in such a way that change in one variable corresponds to change in the other variable then the variables X and Y are correlated.

Karl Pearson's correlation coefficient **is** $r(X,Y) = \dfrac{Cov\ (X,Y)}{\sqrt{Var\,(X)}\ \sqrt{Var\,(Y)}}$

Types: 1. Positive and Negative **2.**Simple,Partial and multiple 3. Linear and Non Linear.

7. **Write any two properties of correlation coefficient.**

Solution: i) Correlation coefficient does not exceed unity.

ii) When r=1 the Correlation coefficient is perfect and positive.

iii) Two independent variables are uncorrelated.

8. **Write any two properties of Regression.**

Solution: i) The regression lines pass through $\left(\bar{x}, \bar{y}\right)$.

ii) When r=1 there is a perfect positive correlation, when r= -1 there is a perfect negative correlation.

9. **Show that** $COV^2(X,Y) \le Var[X]\ Var[Y]$**.**

Solution: Cov (X,Y)=E(XY)-E(X)E(Y)

We know that $[\text{E(XY)}]^2 \le \text{E}(\text{x}^2)E(Y^2)$

$$Cov^2(X,Y) = (E(XY))^2 + [E(X)]^2[E(Y)]^2 - 2E(XY)E(X)E(Y)$$
$$\le E(X^2)E(Y^2) + (E(X^2))(E(Y^2)) - 2E(XY)E(X)E(Y)$$
$$\le E(X^2)E(Y^2) + E(X^2)E(Y^2) - E(X^2)E(Y^2) - E(Y^2)E(X^2)$$
$$\le Var(X).\,Var(Y)$$

10. **If X and Y are independent random variables with variance 2 and 3. Find the variance of 3X+4Y.**

Solution: Given Var(X) = 2 and Var(Y) = 3

Var(3X+4Y) = 3^2 Var(X) +4^2 Var(Y) = 9(2)+16(3)= 66.

11. **If the joint pdf of (X,Y) is** $f(x,y) = \frac{1}{4},\ 0 \le x \le y \le 2$**. Find** $P(X+Y \le 1)$**.**

Solution: $P(X+Y \le 1) = \int_0^1 \int_0^{1-y} f(x,y)\ dx\,dy$

$$= \int_0^1 \int_0^{1-y} \frac{1}{4} dx\,dy = \frac{1}{4}\int_0^1 [x]_0^{1-y} dy$$

$$= \frac{1}{4}\int_0^1 (1-y)\,dy = \frac{1}{4}\left[y - \frac{y^2}{2}\right]_0^1 = \frac{1}{4}\left[1 - \frac{1}{2}\right] = \frac{1}{8}$$

12. **What is the angle between two regression lines?**

Solution: $\tan\theta = \left(\frac{1-r^2}{r}\right)\frac{\sigma_X \sigma_Y}{\sigma_X^2 + \sigma_Y^2}$.

13. **Give any two regression equations (OR) State the equations of the two regression lines.**

Solution: The line of regression of X onY is $x - \bar{x} = r.\frac{\sigma_x}{\sigma_y}(y - \bar{y})$

The line of regression of Y on X is $y - \bar{y} = r.\frac{\sigma_y}{\sigma_x}(x - \bar{x})$

Where r is the correlation coefficient σ_x and σ_y are standard deviation.

14. **If there is no correlation between two random variables X and Y, then what can you say about the regression lines?**

Solution: When there is no linear correlation between X and Y ie) when r_{XY} =0. The equation of the regression lines becomes $y = \bar{y}$ *and* $x = \bar{x}$ which are at right angles.

15. **The two equation of the variable X and Y are x=19.13-0.87y and y=11.64-0.50x. Find the correlation co-efficient between X and Y.(2)**

Solution: Given $b_{XY} = -0.87$, $b_{YX} = -0.50$

$$r = \pm\sqrt{b_{XY}\ b_{YX}} = \pm\sqrt{(-0.87)(-0.50)} = \pm 0.6595$$

16. **The regression lines between two random variables X and Y is given by 3x+y=10 and 3x+4y=12. Find the correlation co-efficient between X and Y.**

Solution: Given 3x+y=10------- (1) 3x+4y=12 ------- (2)

$$(1) \Rightarrow x = \frac{10}{3} - \frac{y}{3}. \quad \therefore b_{XY} = \frac{-1}{3} \qquad (2) \Rightarrow y = 3 - \frac{3}{4}x \quad \therefore b_{YX} = -\frac{3}{4}$$

$$r = \pm\sqrt{b_{XY}\ b_{YX}} = \pm\sqrt{\left(\frac{-1}{3}\right)\left(\frac{-3}{4}\right)} = \pm 0.5$$

17. **The regression equations of X and Y and Y on X are respectively 5x-y=22 and 64x-45y=24. Find the means of X and Y.**

Solution: Since $\bar{x}$ and $\bar{y}$ lies on the given two lines we get

$5\bar{x} - \bar{y}$=22-----------(1), $64\bar{x} - 45\bar{y}$ =24.---------------(2) .

Solving (1) and (2) we get $\bar{x}$=6, $\bar{y}$=8.

18. **The tangent of the angle between the lines of regression y on x and x on y is 0.6 and** $\sigma_x = \frac{1}{2}\sigma_y$. **Find the correlation coefficient.**

Solution: WKT $\tan\theta = \left(\frac{1-r^2}{r}\right)\frac{\sigma_X \sigma_Y}{\sigma_X^2 + \sigma_Y^2}$. Given $\tan\theta = 0.6 = \frac{3}{5}$, $\sigma_x = \frac{1}{2}\sigma_y \Rightarrow \sigma_y = 2\sigma_x$

$$\frac{3}{5}=\left(\frac{1-r^2}{r}\right)\frac{\sigma_X 2\sigma_X}{\left(\sigma_X\right)^2+\left(2\sigma_X\right)^2}\Rightarrow\frac{3}{5}=\left(\frac{1-r^2}{r}\right)\frac{2}{5}\Rightarrow 2r^2+3r-2=0\Rightarrow r=-2,\ \frac{1}{2}$$

$\therefore r=0.5$ [r lies in -1 to +1].

19. **Define one function of two random variables.**

Solution: Let U=u(x,y) be the one function of two random variables. Consider the auxiliary random variable v=y , then the joint pdf of f(u,v) is

$$f(u,v)=|J|\ f(x,y).\ \ where\ \ |J|=\frac{\partial(x,y)}{\partial(u,v)}=\begin{vmatrix}\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v}\\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}\end{vmatrix}$$

The pdf of u is $f(u)=\int_{-\infty}^{\infty} f(u,v)\ d\,v$

20. **Find the distribution function of the random variable Y=g(x), in terms of the distribution function of X, if it is given that** $g(x)=\begin{cases}x-c & for\ \ x>c\\ 0 & for\ \ |x|\le c\\ x+c & for\ \ x<-c\end{cases}$ **.**

Solution: If y<0, $F_Y(y)=P(Y\le y)=P(X+c\le y)=P(X\le y-c=F_X(y-c)$

If $y\ge 0,\ \ F_Y(y)=P(X-c\le y)=F_X(Y+c)$

21. **State Central Limit Theorem.**

Solution: If $x_1,x_2,\ldots x_n$ are n independent identically distributed random variables with same mean μ and standard deviation σ and if $\bar{x}=\frac{1}{n}\sum_{i=1}^{n}x_i$ then the variate $z=\frac{\bar{x}-\mu}{\sigma/\sqrt{N}}$ has a distribution that approaches the standard normal distribution as i $n\to\infty$ provided the m.g.f of x_i exist.

22.The joint p.d.f of (X,Y) is given by $f(x,y)=e^{-(x+y)},0\le x,y\prec\infty$ **are x and y independent?why?**

Solution: To prove: $f(x)f(y)=f(x,y)$

$$f(x)=\int_0^{\infty}e^{-(x+y)}dy=e^{-x}\int_0^{\infty}e^{-y}dy=e^{-x}[\frac{e^{-y}}{-1}]_0^{\infty}=e^{-x}$$

Similarly, $f(y)=\ e^{-y}$

$f(x)f(y)=e^{-x}e^{-y}=e^{-(x+y)}=f(x,y)$ Hence x and y are independent.

23.Is Correlation Coefficient is independent of change of origin and scale?

Solution: Yes. Correlation Coefficient is independent of change of origin and scale.r(X,Y)=r(U,V) where U=$\frac{X-a}{h},V=\frac{y-b}{k}$ a and b are some arbitrary constants.

UNIT III – RANDOM PROCESSES

1. **Define Markov process.**

Solution: A random process X(t) is said to be Markovian if

$$P\left[X(t_{n+1}) \le X_{n+1} \big/ X(t_n), X(t_{n-1}) = X_{n-1}, \ldots X(t_0) = x_0\right] = P\left[X(t_{n+1}) \le X_{n+1} \big/ X(t_n) = X_n\right] \text{ where}$$

$t_0 \le t_1 \le t_2 \le, \ldots \le t_n \le t_{n+1}$. Here $x_0, x_1, x_2, \ldots, x_n, x_{n+1}$ are known as the status of the process.

2. **Define a Markov chain and give an example. (or) What is a Markov chain? When you say that a Markov chain is Homogeneous.**

Solution: Let X(t) be a Markov process which satisfies Markov property and which takes only discrete values whether it is discrete or continuous. Then X(t) is called as Markov chain if

$$P\left[X_n = a_n / X_{n-1} = a_{n-1}, X_{n-2} = a_{n-2}, \ldots, X_0 = a_0\right] = P\left[X_n = a_n / X_{n-1} = a_{n-1}\right] \;\; \forall n.$$

Homogeneous chain: If the Markov chain is Homogeneous then $P_{ij}(n-1, n) = P_{ij}(m-1, m)$.

3. **What is meant by steady state distribution of Markov chain?**

Solution: If the homogeneous Markov chain is regular, then every sequence of state probability distribution approaches a unique fixed probability distribution.

4. **State the four types of random processes (or) Stochastic process.**

Solution: i) Discrete Random Process ii) Continuous Random Process

iii) Discrete Random Sequence iv) Continuous Random Sequence

5. **State any four properties of Poisson process.**

Solution: i) Poisson process is a Markov process.

ii) The sum of two independent Poisson process is again a Poisson process

iii) The difference of two Poisson process is not a Poisson process

iv)The inter arrival time of a Poisson process with parameter has an exponential distribution with mean $\frac{1}{\lambda}$.

6. **What is a stationary process? (or) What do you understand by stationary process?**

Solution: Stationary Process: In certain probability distribution or averages do not depend on time t then the random process {X(t)} is called a Stationary Process.

Non-stationary process: In certain probability distribution or averages depend on time t then the random process {X(t)} is called a Non Stationary Process.

7. **Define Strict sense Stationary processes and Wide-sense Stationary Process.**

Solution: Strict sense Stationary processes: A random process {X(t)} is called a **SSS** process if all its finite dimensional distributions are invariant under translation of the time parameter.

Wide-sense Stationary Process: A random process {X(t)} is called a **WSS** process if

i) Mean is constant ii) Auto correlation function depends only on the time difference τ .

8. **Define a birth process.**

Solution: Let X(t) represents the number of individuals aline at time t in a population. Two types of event can occur- Birth and Death. If there is an increase in population, then it is known as Birth Process.

9. **Distinguish between wide sense Stationary and strict sense Stationary of the random process X(t).**

Solution:

S.No.	WSS Process	SSS Process
1	Every SSS process of order 2 is a WSS process	Every WSS process of order 2 is a SSS process
2	E[X(t)]=Constant. The condition is not enough	E[X(t)]=Constant. The condition is enough.

10. **If** $\{X(s,t)\}$ **is a random process, what is the nature of X(s,t) when a) s is fixed, b) t is fixed?**

Solution: a)when s is fixed $\{X(s,t)\}$ is a time function.

b) when t is fixed $\{X(s,t)\}$ is a random variable

11. **If X(t) is a stationary process,** $R(t_1,t_2)=E\left\{X(t_1)X(t_2)\right\}$ **is a function of** (t_1-t_2), **what are** $E\left\{X^2(t)\right\}$ *and* $Var\left\{X(t)\right\}$**.**

Solution: $E\left\{X^2(t)\right\}=Cons\tan t$ *and* $Var\left\{X(t)\right\}=Cons\tan t$.

12. **State Chapman-Kolmogorow theorem.**

Solution: If P is the TPM of a homogeneous Markov chain, then the n-step tpm is $\left[P_{ij}{}^{(n)}\right]=\left[P_{ij}\right]^n$

13. **What is a stochastic matrix? When is it said to be regular.**

Solution: If $P_{ij}>0$ *and* $\sum P_{ij}=1$ for all i, then the TPM of a Markov Chain is a **stochastic matrix P.** A stochastic matrix P is said to be regular matrix, if all the entries of P^m are positive

14. **What is meant by one-step transition probability ?**

Solution: The conditional probability $P\left\{X_n=a_j/X_{n-1}=a_i\right\}$. ie) From state a_i to state a_j at the n^{th} step and is denoted by i $P_{ij}(n-1,n)$.

15. **Define irreducible and reducible matrix.**

Solution: Irreducible Matrix: If $P_{ij}^{(n)} > 0$, for some n and for every i and j, then every state can be reached from every other state. Then the Markov chain is said to be **irreducible.** The TPM of an irreducible chain is an **irreducible matrix.** Otherwise the chain is said to be **reducible.**

16. **Define return state, Periodic and Aperiodic.**

Solution: Return State: If $P_{ij}^{(n)} > 0$, for some n>1, then we call the state i of the Markov chain as return state.

Periodic: Let $P_{ii}^{(m)} > 0$ for all m. Let I be a return state, then we define the period d_i as follows

$$d_i = GCD\left\{m : P_{ii}^{(m)} > 0\right\}.$$

APeriodic: If d_i=1 then the state i is said to be aperiodic. If d_i>1, then the state i is said to be periodic.

17. **Define Poisson process (or) State the postulates of Poisson process.**

Solution: If X(t) represents the number of occurrences of a certain event in (0,t), then the discrete random process {X(t)} is called the Poisson process provided the following postulates are satisfied

$i)\, P[1 \text{ occurrence in } (t, t+\Delta t)] = \lambda \Delta t + o(\Delta t)$

$ii)\, P[0 \text{ occurrence in } (t, t+\Delta t)] = 1 - \lambda \Delta t + o(\Delta t)$

$iii)\, P[2 \text{ occurrence in } (t, t+\Delta t)] = o(\Delta t)$

iv)X(t) is independent of the number of occurrences of the event in any interval (0,t).

18. **What will be the superposition of n independent Poisson processes with respective average rates $\lambda_1, \lambda_2, ..., \lambda_n$.**

Solution: The superposition of n independent Poisson processes with respective average rates $\lambda_1, \lambda_2, ..., \lambda_n$ is another Poisson process with average rate $\lambda_1 + \lambda_2 + ... + \lambda_n$.

19. **Find the Mean of a Poisson process. Examine whether it is stationary or not. (or) Show that the Poisson process is not covariance stationary.**

Solution: WKT $P_X(t) = \dfrac{(\lambda t)^x e^{-\lambda t}}{x!}$

Mean = E[X(t)]= $= \sum_{x=0}^{\infty} x P_X(t)$

$$= \sum_{x=0}^{\infty} x \frac{(\lambda t)^x e^{-\lambda t}}{x!} = e^{-\lambda t} \sum_{x=0}^{\infty} x \frac{(\lambda t)^x}{x(x-1)!}$$

$$= e^{-\lambda t}\left[\frac{\lambda t}{0!} + \frac{(\lambda t)^2}{1!} + \frac{(\lambda t)^3}{2!} + ...\right] = e^{-\lambda t} \lambda t \left[1 + \frac{\lambda t}{1!} + \frac{(\lambda t)^2}{2!} + ..\right] = e^{-\lambda t} \lambda t e^{\lambda t}$$

$$\therefore Mean = \lambda t$$

Since the Mean depend on t, the Poisson process is not a stationary process.

20. **Consider the random process $X(t) = \cos(t+\phi)$ where ϕ is uniform in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Check whether the process is stationary.(2)**

Solution: If a random process is stationary then E[X(t)]=constant.

Given $X(t) = \cos(t+\phi)$ and $f(\phi) = \frac{1}{\pi},\ -\frac{\pi}{2} < \phi < \frac{\pi}{2}$

$$\therefore E\left[X(t)\right] = \int_{-\infty}^{\infty} X(t)\, f(\phi)\, d\phi$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(t+\phi) \frac{1}{\pi} d\phi = \frac{1}{\pi}\left[\sin(t+\phi)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{1}{\pi}\left[\sin(t+\frac{\pi}{2}) - \sin(t-\frac{\pi}{2})\right]$$

$$= \frac{1}{\pi}\left[\sin(\frac{\pi}{2}+t) + \sin(\frac{\pi}{2}+t)\right] \qquad [\sin(90+\theta) = \sin(90-\theta) = \cos\theta]$$

$$= \frac{2}{\pi}\cos t \neq Cons\tan t \quad [\because It\ is\ a\ function\ of\ t]$$

Hence X(t) is not a stationary process.

21. **Consider the random process $X(t) = \cos\left(\omega_0 t + \theta\right)$, where θ is uniformly distributed in the interval $-\pi$ to π. Check X(t) is stationary or not.Solution:**Here E(X(t)) is constant and Var(X(t)) is also constant,hence X(t) is Stationary.

22. **Find the autocorrelation function of a Poisson process X(t) with rate λ. (or) Let X(t) be a Poisson process with rate λ. Find the correlation function of X(t).**

Solution: Let $R_{XX}(t_1, t_2)$ is defined as the autocorrelation at time interval (t_1,t_2).

$$R_{XX}(t_1,t_2) = E[X(t_1)X(t_2)] = E[X(t_1)\{X(t_2) - X(t_1) + X(t_1)\}]$$

$$= E[X(t_1)]E[X(t_2) - X(t_1)] + E[X^2(t_1)]$$

$$= \lambda t_1 . \lambda(t_2 - t_1) + (\lambda t_1)^2 + \lambda t_1$$

$$= \lambda^2 t_1 t_2 + \lambda \min(t_1, t_2)$$

23. **State and prove additive property of Poisson process. (or) The additive property holds good for any number of independent Poisson processes. Justify.**

Solution: Statement : The sum of two independent Poisson Process is again a Poisson Process.

Proof: Let $\{X_1(t), t \geq 0\}, \{X_2(t), t \geq 0\}$ be two independent Poisson process, then their MGF is

$M_{X_1(t)}(\omega) = e^{-\lambda_1 t\left(1-e^{-\omega}\right)}$ *and* $M_{X_2(t)}(\omega) = e^{-\lambda_2 t\left(1-e^{-\omega}\right)}$. Since X_1(t) and X_2(t) are independent then

$$M_{X_1(t)}(\omega)\, M_{X_2(t)}(\omega) = e^{-\lambda_1 t\left(1-e^{-\omega}\right)}\ e^{-\lambda_2 t\left(1-e^{-\omega}\right)}$$

$$M_{X_1(t+)X_2(t)}(\omega) = e^{-(\lambda_1+\lambda_2)\ t\left(1-e^{-\omega}\right)}$$

which is the MGF of Poisson process with parameter $(\lambda_1 + \lambda_2)\ t$.

Hence the sum of two independent Poisson Process is again a Poisson Process.

$\therefore$ The additive property holds good for any number of independent Poisson processes.

24. **If the transition probability matrix of a Markov chain is $\begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix}$. Find the limiting distribution (or steady state distribution or long run) of the Chain?(2)**

Solution: The limiting distribution of the Chain is given by the property of π . Let $\pi = (\pi_1, \pi_2)$ be the stationary state distribution of the Markov chain.

$$\therefore (\pi_1 \quad \pi_2)\begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix} = (\pi_1 \quad \pi_2)$$

$$\frac{\pi_2}{2} = \pi_1 ---(1) \quad and \quad \pi_1 + \frac{\pi_2}{2} = \pi_2 ---(2)$$

Since π is a stationary distribution. $\therefore \pi_1 + \pi_2 = 1$

Solving (1) and (2) we get $\pi_1 = \frac{1}{3}$ and $\pi_2 = \frac{2}{3}$

25. **If particles are emitted from a radio active source at the rate of 20 per hour, find the probability that exactly 5 particles are emitted during a 15 minute period.**

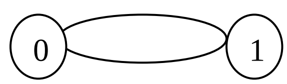
Solution: Given $\lambda = 20/hr$, $t = 15\,\text{min} = \frac{15}{60} = \frac{1}{4}/hr$

Poisson Process is $P[X(t) = n] = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$ $\quad \therefore P[X(t) = 5] = \frac{e^{-20\frac{1}{4}}\left(20\frac{1}{4}\right)^5}{5!} = 0.17448$

26. **The one step transition probability matrix of a Markov chain with states (0,1) is given by $P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. a)Draw the transition diagram b)Is it irreducible Markov Chain? (2)**

Solution:

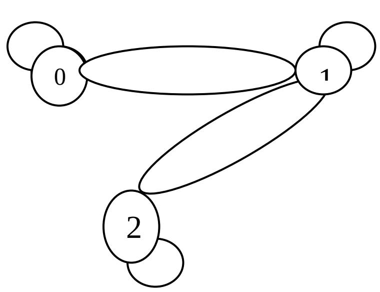
a) Ttransition diagram



b) Since every state is reached from every other state. The Matrix is irreducible.

27. **Determine whether the given matrix is irreducible or not** $P = \begin{bmatrix} 0.3 & 0.7 & 0 \\ 0.1 & 0.4 & 0.5 \\ 0 & 0.2 & 0.8 \end{bmatrix}$.

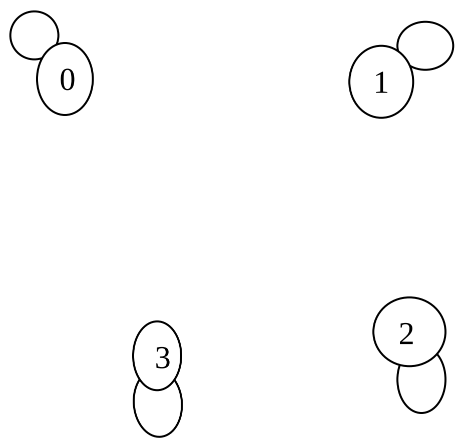
Solution: Since there is no accessible between 0 and 2 and hence the chain is non-irreducible.



28. **Consider the Markov chain with transition probability matrix:** $\begin{bmatrix} 0.4 & 0.6 & 0 & 0 \\ 0.3 & 0.7 & 0 & 0 \\ 0.2 & 0.4 & 0.1 & 0.3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$**. Is it irreducible? If not, find the classes. Find the nature of states.**

Solution: Here the Markov chain has 4 states {0,1,2,3}.

The transition diagram is



From the figure it is obvious no other state is accessible from state 3 and P_{33}=1. Hence state 3 is an absorbing state. Since every state is not reachable from any other state, the Markov chain is not irreducible. The classes of the chain are {0,1}, {2}, {3}.

29. **Consider a Markov chain** $\left\{X_n;\ n=0,1,2,...\right\}$ **with state space** $S=\left\{1,2\right\}$ **and one step transition probability matrix** $P=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$**. Is the state 1 is periodic?**

Solution: Given $P=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

$P^2 = P.P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\qquad$ $P^3 = P^2.P = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$P^4 = P^3.P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\qquad$ $P^5 = P^4.P = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

To find Period: Wkt $d_i = GCD\left\{m : P_{ii}^{(m)} > 0\right\}$. Since $P^{2n} = P^2$ ie) $P_{ii}^{(2)} = P_{ii}^{(4)} = P_{ii}^{(6)} = ... > 0$.

$\therefore GCD\ of\ \left\{2,\ 4,\ 6, ...\right\} = 2$. Hence the state 1 is periodic with period 2.

30.P.T the difference of two independent Poisson process is not a Poisson Process.

Solution: $E[X(t)]=E[X_1(t)-X_2(t)]=(\lambda_1 - \lambda_2)t$

$E[X^2(t)] = (\lambda_1 - \lambda_2)^2 t^2 + (\lambda_1 + \lambda_2)t \neq (\lambda_1 - \lambda_2)^2 t^2 + (\lambda_1 - \lambda_2)t$

Hence $X(t) = X_1(t) - X_2(t)$ is not a Poisson process.

UNIT IV-CORRELATION AND SPECTRAL DENSITIES

1.Define Auto correlation function.

Solution: If the process $\{X(t)\}$ is either wide sense stationary or strict sense stationary then $E\{X(t)\,X(t+\tau)\}$ is a function of τ, denoted by $R_{xx}(\tau)$. This function $R_{xx}(\tau)$ is called the Auto correlation function of the process $\{X(t)\}$.

2. Prove that $R_{xx}(\tau)$ is an even function of τ.

Solution: We know that $R_{xx}(\tau) = E[X(t)X(t+\tau)]$

similarly $R_{xx}(-\tau) = E[X(t)X(t-\tau)]$

put $t-\tau = P \Rightarrow t = P+\tau$

$\therefore R_{xx}(-\tau) = E[X(P+\tau)X(P)]$

$= E[X(P)X(P+\tau)]$

$= R_{xx}(\tau)$

3.Define Cross-correlation function and state any two of it's properties.When do you say that they are independent?

Solution: Let $\{X(t)\}$ and $\{Y(t)\}$ be two random processes.Then the cross correlation between them is defined as $R_{XY}(t,t+\tau) = E[X(t)Y(t+\tau)] = R_{XY}(\tau)$.

If the random process X(t) and Y(t) are independent then $R_{XY}(\tau) = E(X).E(Y)$

Properties: (i) $R_{XY}(\tau) = R_{YX}(-\tau)$

(ii) $|R_{XY}(\tau)| \leq \sqrt{R_{XX}(0) + R_{YY}(0)}$

4.State any two properties of an auto correlation function.

Solution: $R(\tau)$ is an even function of τ.

$R(\tau)$ is maximum at $\tau = 0$.

5.The power spectral density of a random process {X(t)} is given by $S_{XX}(\omega) = \begin{cases} \Pi, & |\omega| \prec 1 \\ 0, & elsewhere \end{cases}$ Find it's Auto correlation function.

Solution: $R_{XX}(\tau) = \dfrac{1}{2\Pi}\int_{-\infty}^{\infty} S_{XX}(\omega) e^{i\omega\tau} d\omega$

$$= \frac{1}{2\Pi}\int_{-1}^{1} \Pi e^{i\omega\tau} d\omega$$

$$= \frac{1}{2}\left[\frac{e^{i\omega\tau}}{i\tau}\right]_{-1}^{1}$$

$$= \sin\tau / \tau$$

6.Define spectral density.

Solution: Let $\{X(t), t \geq 0\}$ be a stationary time series with $E[X(t)]=0$ and covariance function $R(t-s)=E[X(t)X(s)]$ and let $F(x)$ be a real never decreasing and bounded function of x with $dF(x)=f(x)dx$. $R(t)$ is non-negative definite then $f(w) = \frac{1}{2\Pi}\int_{-\infty}^{\infty} R(\tau)\, e^{i\omega\tau}\, d\tau$ which is called spectral density.

7. Define power spectral density function

Solution: The power spectral density $S_X(\omega)$ of a continuous time random process X(t) is defined as the Fourier transform of $R_{xx}(\tau)$:

$$S_{XX}(\omega)=\int_{-\infty}^{\infty} R_{xx}(\tau)\, e^{-i\omega\tau} d\tau \text{ --------------- } (1)$$

Taking the inverse Fourier transform of $S_{XX}(\omega)$ we obtain

$$R_{XX}(\tau)=\frac{1}{2\Pi}\int_{-\infty}^{\infty} S_{XX}(\omega)\, e^{i\omega\tau}\, d\omega \text{ ------- } (2)$$

8.Find the power spectral density function of the stationary process whose auto correlation function is given by $e^{-|\tau|}$.

Solution: $R_{XX}(\tau)= e^{-|\tau|}$

$$S_{XX}(\omega)=F[R_{XX}(\tau)]=F[e^{-|\tau|}] = \frac{2}{1+\omega^2}$$

9. Find the variance of the stationary process {X(t)}, whose auto correlation function is given by $R_{XX}(\tau) = 2 + 4e^{-2|\tau|}$.

Solution: $R_{XX}(\tau) = 2 + 4e^{-2|\tau|}$

$\lim_{\tau-\infty} R(\tau) = 2 + 4e^{-2\infty} = 2 + 0 = 2$

$E(X(t))=+\sqrt{2} or -\sqrt{2}$

$E(X^2_{(t)})=R(0)=2+4e^{-0}=6$

$Var[X(t)]=R(0)-R(\infty)=6-2=4$

10.State any two properties of cross correlation function.

Solution:(i)$R_{XX}(\tau)=R_{YX}(-\tau)$

(ii)If X(t) and Y(t) are two random processes and $R_{XX}(\tau)$ and $R_{YY}(\tau)$ are their respective auto correlation functions then $|R_{XY}(\tau)| \leq \sqrt{R_{XX}(0) R_{YY}(0)}$

11. The auto correlation function of a stationary random process is $R(\tau) = 16 + \frac{9}{1+16\tau^2}$.Find the mean and variance of the process.

Solution: $R(\tau)=16+\frac{9}{1+16\tau^2}$

$\lim_{\tau-\infty} R(\tau)=16+\frac{9}{1+16\infty^2}=16+0=16$

E(X(t))= $\sqrt{16}$ = ± 4

E($X^2{}_{(t)}$)=R(0)=16+ $\frac{9}{1+0}$ =25

Var[X(t)]=R(0)-R(∞)=25-16=9

12. The auto correlation function of a stationary random process is $R_{XX}(\tau)=25+\frac{4}{1+6\tau^2}$ **.Find the mean and variance of the process.**

Solution: $R(\tau)=25+\frac{4}{1+6\tau^2}$

$\lim_{\tau-\infty} R(\tau)=25+\frac{4}{1+6\infty^2}=25+0=25$

E(X(t))= $\sqrt{25}$ = ± 5

E($X^2{}_{(t)}$)=R(0)= 25+ $\frac{4}{1}$ =29

Var[X(t)]=R(0)-R(∞)=29-25=4

13.State any 2 properties of cross power density spectrum.

Solution :(i) $S_{xy}(\omega)=S_{yx}(-\omega)$

(ii) $S_{XY}(\omega)$ if X(t) are orthogonal.

14.A random process X(t) is defined by $X(t)=K\cos\omega t, t\geq 0$ **where** ω **is a constant and K is uniformly distributed over (0,2).Find the auto correlation function of X(t).**

Given k is a r.v uniformly distributed over (0,2)

Solution: f(k)={ $\frac{1}{2-0} in(0,2)$

$0, otherwise$

$X(t)=K\cos\omega t, t\geq 0$

$R_{XX}(\tau)$=E[(X(t)X(t+τ)] (by definition)

=E[(kcos $\omega t)(k\cos\omega(t+\tau)]$

=E[$k^2(\cos\omega t\cos\omega(t+\tau)$]

= $\cos\omega t\cos\omega(t+\tau)$ E[k^2]

= $\cos\omega t\cos\omega(t+\tau)\int_0^2 k^2 f(k)dk$

= $\cos\omega t\cos\omega(t+\tau)\int_0^2 k^2\frac{1}{2}dk$

$$= \frac{1}{2}\cos\omega t\cos\omega(t+\tau)\left(\frac{8}{3}\right)$$

$$= \frac{4}{3}\cos\omega t\cos\omega(t+\tau)$$

15.Define cross correlation function of X(t) andY(t).

Solution:Let {X(t)} and {Y(t)} be two random processes. Then the cross correlation between them is defined as

$$R_{XY}(t,t+\tau) = E[X(t)Y(t+\tau)] = R_{XY}(\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xyf(x,y:t,t+\tau)dxdy$$

16.State WIENER-KHINTCHINE theorem.

Statement: If $X_T(\omega)$ is the Fourier transform of the truncated random process is defined as

$X_T(t)= \begin{cases} X(t), \ for|t| \le T \\ 0, \ for|t| \succ T \end{cases}$ where [X(t)] is a real WSS process with power spectral density functions(ω) then

$S(\omega)=\lim_{t\to\infty}[\frac{1}{2T}E\{|X_T(\omega)|^2\}]$

17.Give an example of Cross spectral density.

Solution:$S_{XX}(\omega)= \begin{cases} a+jb\omega, |\omega|<1 \\ 0, elsewhere \end{cases}$

18. **The auto correlation function of a stationary process with no periodic components is $R_{XX}(\tau) = 36 + \frac{9}{1+8\tau^2}$ .Find the mean value of the process X(t).**

Solution: Given $R_{XX}(\tau) = 36 + \frac{9}{1+8\tau^2}$

We know that $\lim_{\tau\to\infty} R_{XX}(\tau) = [\bar{X}]^2$

$$[\bar{X}]^2 = \lim_{\tau\to\infty}[36 + \frac{9}{1+8\tau^2}] = 36 + \frac{9}{\infty} = 36 + 0 = 36 \Rightarrow \bar{X} = 6.$$

19.State any two properties of spectrum.

Solution: (i) For a wide sense stationary random process,power spectral density at zero frequency gives the area under the graph of Auto Correlation.

(ii)The mean square value of a wide-sense stationary process is equal to the total area under the graph of Spectral density.

20.Write any two uses of Wiener-Khinchine theorem.

Solution:(i) This theorem provides an alternative method for finding S(ω) for a WSS process.

(ii)It relates time and frequency characteristics of a random process.

(iii)Computing power spectrum of a random process.

UNIT: V LINEAR SYSTEMS WITH RANDOM INPUTS

1.Define time –invariant system.

Solution: If Y(t+h) = f[X(t+h)] where Y(t) = f[X(t)] then f is called a time-invariant system.

2.Define a system. When it is called a linear system?

Solution: System: Mathematically a sysyem is a functional relationship between the input $X(t_0)$ and output Y(t). The input and output relationship can be written as $Y(t_0)=f[x(t):-\infty \prec t \prec \infty];-\infty \prec t_o \prec \infty$.

Then for linear system $f[a_1x_1(t)+a_2x_2(t)]=a_1Y_1(t)+a_2y_2(t)$.

3.If the input function X(t) and it's output Y(t) are related by of Y(t)= $\int_{-\infty}^{\infty} h(u)X(t-u)du$ **then prove that the system is a linear time invariant system.**

Solution: Given that $Y(t)=\int_{-\infty}^{\infty} h(u)X(t-u)du$

i) $Y(t)=\int_{-\infty}^{\infty} h(u)[a_1X_1(t-u)+a_2X_2(t-u)]du$

$=a_1Y_1(t)+a_2Y_2(t) \Rightarrow Y(t)$ is linear

ii) $Y(t) = \int_{-\infty}^{\infty} h(u)X(t-u)du$

$Y(t)=\int_{-\infty}^{\infty} h(u)X(t+k-u)du$

=Y(t+k) (X(t) is replaced by X(t+k)

$\Rightarrow$Y(t) is time invariant.

4.State any two properties of a linear time- invariant system.

Solution: (i) $S_{yy}(\omega)=|H(\omega)|^2 S_{xx}(\omega)$

(ii) If the input X(t) and its output Y(t) are related by $Y(t)=\int_{-\infty}^{\infty} h(u)X(t-u)du$ then the system is a linear time-invariant system.

5.**State the properties of linear filter**.

Solution: Let $\{X_1(t)\}$ and $\{X_2(t)\}$be any two processes and a and b be two constants.If L is a linear filter then $L[aX_1(t)+bX_2(t)]=a\,L[X_1(t)]+bL[X_2(t)]$

6.If $\{X(t)\}$ **and** $\{Y(t)\}$ **in the system** $Y(t)=\int_{-\infty}^{\infty} h(u)X(t-u)du$ **are WSS process, how are their auto correlation function related**.

Solution:Given $Y(t) = \int_{-\infty}^{\infty} h(u)X(t-u)du$

We know that $R_{XY}(\tau)=E[X(t)Y(t+\tau)$

$=E[X(t)\int_{-\infty}^{\infty} h(u)X(t+\tau-u)du$

$=\int_{-\infty}^{\infty} R_{XX}(\tau-u)h(u)du$ since X(t) is a wss)

$R_{XY}(\tau)=R_{XX}(\tau)*h(\tau)$

7.Define a linear system with an random input.

Solution: We assume that x(t) represents a sample function of a random process {X(t),the system produces an output or response y(t) and the ensemble of the output functions forms a random process {y(t)}. The process {y(t)} can be considered as the output of the system or transformation f with {X(t)} as the input,the system is completely specified by the operator f.

8.What is unit impulse response of a system? Why is it called so?

Solution: $\int_{-\infty}^{\infty} \Phi(t)\delta(t-a)dt = \Phi(a)$ where $\delta(t-a)$ is the unit impulse function at a .If we take a=0 and X(t)= $\delta(t)$ then Y(t)=h(t).Thus if the input of the system is the unit impulse function,then the output or response is the system weighing function.Therefore the system weighing function h(t) will be called unit impulse response function.

9. Define a linear time –invariant system.

Solution: If x(ω),y(ω) and H(ω) are the respective fourier transforms of x(t),y(t) and n(t) then Y(ω)= $\int_{-\infty}^{\infty} y(t)e^{-i\omega t}dt = x(\omega)H(\omega)$ where H(ω) is called the transfer function of the system.The above relation show that the response of any linear time-invariant system.

10.State the convolution form of the output of a linear time –invariant system.

Solution: $R_{XY}(\tau) = R_{XX}(\tau) * h(\tau)$

$R_{YY}(\tau) = R_{XY}(\tau) * h(-\tau)$

$S_{XY}(\omega) = S_{XX}(\omega) * H(\omega)$,where * denotes convolution.

11.A system has an impulse response h(t)= $e^{-\beta t}u(t)$ **find the power spectral density of the output y(t) corresponding to the input x(t).**

Solution: h(t)= $e^{-\beta t}u(t)$

$\Rightarrow$H(ω)=F[h(t)]=F[$e^{-\beta t}u(t)$ **]=** $\dfrac{1}{\beta + i\omega}$

$\Rightarrow |H(\omega)|^2 = \dfrac{1}{\beta^2+\omega^2}$

we know that $S_{yy}(\omega) = |H(\omega)|^2 S_{XX}(\omega)$

$\Rightarrow S_{yy}(\omega) = |H(\omega)|^2 S_{XX}(\omega)$

$\Rightarrow S_{yy}(\omega) = \dfrac{1}{\beta^2 + \omega^2} S_{XX}(\omega)$

12.Give an example for a linear system.

Solution: A signal process Using signal x(t) as input we could obtain a specific y(t) as output.

PART - B

UNIT I - RANDOM VARIABLES

1. A random variable X has the following probability function

Values of X	0	1	2	3	4	5	6	7	8
P(x)	a	3a	5a	7a	9a	11a	13a	15a	17a

i)Determine the value of a. ii) Find $p(X<3),\ p(X\geq 3),\ p(0<X<5)$.
iii) Find the distribution function of X.

2. A random variable X ha the following probability function

X	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	K^2	$2K^2$	$7K^2+k$

i)Determine the value of k. ii) Find $p(X<6),\ p(X\geq 6),\ p(0<X<5)$.
iii) Find $p(1.5<X<4.5/X>2)$ iv)Find the distribution function of X.

3. The density function of continuous random variable X is given by

$$f(x)=\begin{cases} ax\ , & 0\leq x<1, \\ a\ , & 1\leq x<2, \\ 3a-ax\ , & 2\leq x<3, \\ 0\ , & otherwise. \end{cases}$$

i) Find the value of a. ii) Cumulative distribution of X.

4. A cumulative distribution function is given by $F(x)=\begin{cases} 0, & x<0, \\ x^2, & 0\leq x<\frac{1}{2}, \\ 1-\frac{3}{25}(3-x^2), & \frac{1}{2}\leq x<3, \\ 1, & x\geq 3. \end{cases}$

Find i) PDF of X. ii) $p(\frac{1}{3}\leq x<4)$. iii) $p(|x|\leq 1)$.

5. Let X be a discrete RV whose cumulative distribution function is

$$F(x)=\begin{cases} 0, & for\ x<-3, \\ 1/6, & for\ -3\leq x<6, \\ 1/2, & for\ 6\leq x<10, \\ 1, & for\ x\geq 10. \end{cases}$$

Find i) PDF of X. ii) $P(X\leq 4),\ P(-5<X\leq 4)$.

6. A continuous random variable X has the PDF $f(x)=3x^2,\ 0<x<1$. Find a and b such that $i)P(X\leq a)=P(X>a).\quad ii)P(X>b)=0.05$

7. A random variable X has the following probability function Find K and the mean of X.

X	-1	0	1	2
P(X=x)	0.4	k	0.2	0.3

8. A random variable X has the following probability function

X	-1	0	1	2
P(X=x)	1/3	1/6	1/6	1/3

Find

$i)\ E(X)\ ii)\ E(X^2)\ iii)\ E(2X+3)^2\ iv)\ Var(X)$

9. A continuous random variable X has the PDF $f(x) = Kx(2-x),\ 0 \le x \le 2$.Find i) K, ii) Mean and iii) Var (X).

10. The density function of continuous random variable X is given by $f(x) = \begin{cases} Ae^{-x}, & 0 < x < \infty, \\ 0, & otherwise. \end{cases}$

i) Find the value of A. ii) r^{th} moment about origin iii) 3^{rd} moment about mean.

11.Find MGF of $f(x) = \begin{cases} x, & 0 < x \le 1, \\ 2-x, & 1 \le x < 2, \\ 0, & otherwise. \end{cases}$ hence find mean and variance.

12.Let the random variable X have the p.d.f $f(x) = \begin{cases} \frac{1}{2}e^{-\frac{x}{2}}, & x > o \\ 0, & otherwise. \end{cases}$. Find the MGF, Mean and Variance of X.

13.i) .The p.d.f of a continuous random variable X is $f(x) = Ke^{-|x|}$ Find K and the F[X].

13.ii)A continuous random variable X has the PDF $f(x) = \begin{cases} Cxe^{-x}, & x > o \\ 0, & x \le 0. \end{cases}$ Find the value of C. Obtain the MGF, mean and variance.

14.A continuous random variable X has the PDF $f(x) = Kx^2e^{-x},\ x \ge 0.$ Find the r^{th} moment of X about the origin. Hence find the first four moments about mean, mean and variance.

15.A random variable X has the probability function $P(x) = \frac{1}{2^x},\ x = 1, 2, 3, \ldots$ Find MGF, Mean and variance.

15.i) The probability function of an infinite discrete distribution is given by $P(X = j) = \frac{1}{2^j}(j = 1, 2, 3.....)$

Find(i)Mean of Xii)P(X is even)iii)P(X is divisible by 3)

16.If the moments of a random variable X are defined by $E(X^r) = 0.6;\ r = 1, 2, 3, \ldots.$ show that $P(X = 0) = 0.4, P(X = 1) = 0.6,\ P(X \ge 2) = 0.$

17.The random variable X has the MGF $M_X(t) = \frac{2}{2-t}$, Find mean and variance ox X.

18. Derive the MGF, mean and variance of binomial distribution with parameters n and p.

19.The mgf of x is given by $M_X(t) = (0.6e^t + 0.4)^8$ find the variance of x and mgf of Y=3X+4.

20.In a large consignment of electric bulbs 10% are defective. The random sample of 20 is taken for inspection. Find the probability that i) All are good bulbs. ii) Atmost there are 3 defectives. iii) Exactly there are 3 defectives.

21.A machine manufacturing screws is known to produce 5% defective. In a random sample of 15 screws, what is the probability that there are i) Exactly 3 defectives. ii) Not more than 3 defectives.

22. Out of 800 families with four children each, How many family would be expected to have (i) two boys and two girls. ii) atleast one boy iii) atmost 2 g irls . Asssume that equal probability for boys and girls.

23. If X is a binomial distributed R.V with E(X)=2 and Var(X)=4 / 3, find P(X=5).

24. Derive the MGF, Mean and Variance of Poisson distribution with parameter λ.

25. Derive the probability mass function of Poisson distribution as a limiting case of Binomial distribution.

26. Six coins are tossed 6400 times using Poisson distribution what is the approximate probability of getting 6 heads in 10 times.

27.A number of monthly breakdown of a computer is a random variable X having a Poisson distribution with mean 1.8. Find the probability that this computer will function for a month i) without breakdown ii) with only one breakdown iii) atleast one breakdown.

28. Find the MGF of Geometric random variable with pdf $P(X = x) = pq^{x-1}, x = 1,2,3,...$ and hence obtain its mean and variance.

29.Establish the memory less property of geometric distribution

30.A die is cast until 6 appears. What is the probability that it must be cast more than 5 (or 4)times?

31.Derive the MGF, Mean and Variance of negative binomial distribution.

32.Suppose that X is a negative binomial random variable with p=0.2 and r=4. Determine the mean of X.

33.Derive the MGF , mean and variance of uniform distribution.

34. A random variable X has a uniform distribution over the interval $(-3,3)$. Find

$$i) P(X < 2),\ ii) P(|X| < 2),\ iii) P(|X - 2| < 2),\ iv) Find\ K\ such\ that\ P(X > K) = \frac{1}{3}.$$

35. If X is uniformly distributed over $(-\alpha,\alpha),\ \alpha > 0.$ *Find* α so that

$$i) P(X > 1) = \frac{1}{3},\ ii) P(|X| < 1) = P(|X| > 1)$$

36.Buses arrive at a specified stop at 15min intervals starting at 8A.M., that is they arrive at 8, 8.15,8.30 and soon. If the passenger arrives at the stop at a random time that is Uniformly distributed between 8 and 8.30A.M., find the probability that he waits (i) less than 5 min for a bus (ii)atleast 12 min for a bus.

37. Derive the MGF, mean and variance of Exponential distribution.

38.Establish the memory less property of Exponential distribution.

39. The mileage which car owners get with a certain kind of radial type is a random variable having an exponential distribution with mean 40.000km. Find the probabilities that one of these tires will last i)atleast 20.000km. ii) atmost 30,000km.

40.The time (in hours) required to repair a machine is exponential distribution parameter $\lambda = \frac{1}{2}$.

What is the i) probability that repair time exceed 2 hours. ii) conditional probability that a repair time takes atleast 10hours given that its duration exceeds 9 hours.

41.The daily consumption of milk in excess of 20,000 liters in a town is approximately exponentially distributed with parameter $\lambda = \frac{1}{3000}$. The town has daily stock of 35,000 liters. What is the probability that of 2 days selected at random the stock is insufficient for both days.

42.Derive the MGF, mean and variance of Gamma distribution.

43. If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.8.What is the probability that he will finally pass the test i) On the fourth trial ii) In lessthan 4 trials?

44. A random variable X is uniformly distributed over (0,10). Find
i)P(X<3),P(X>7) and P(2<X<5)
ii)P(X=7)

45. If X and Y are independent random variables following N(8,2) and N(12,4 $\sqrt{3}$) respectively, find the value of λ such that $P(2X - Y \le 2\lambda) = P[X + 2Y \ge \lambda]$.

46.Let X and Y be independent normal variates with mean 45 and 44 and standard deviation 2 and 1.5 respectively. What is the probability that randomly chosen values of X and Y differ by 1.5 or more?

47. Given that X is distributed normally , ifP[X<45}=0.31 and P[X>64]
Find the mean and standard deviation of the distribution.

48. The marks obtained by a number of students in a certain subject are assumed to be normally distributed with mean 65 and standard deviation 5. If 3 students are selected at random from this group, what is the probability that two of them will have marks over 70?

UNIT II

TWO DIMENSIONAL RANDOM VARIABLES

1.The joint pdf of X and Y is given by $f(x, y) = \frac{x + y}{21}$, $x = 1, 2, 3.\ y = 1, 2.$ Find the marginal distributions

2.The two dimensional random variable (X,Y) has the joint density function $f(x, y) = \frac{x + 2y}{27}, x = 0,1,2.$ Find the conditional distribution of Y given X=x.Also find the conditional distribution of X given Y=1.

3.If the joint pdf of (X,Y) is given by P(x,y)=k(2x+3y) , x=0,1,2. y=1,2,3. Find all the mariginal distridutios and the conditional distributions. Also find the probability distribution of (X+Y) and P(X+Y>3).

4.Three balls are drawn at random without replacement from a box containing 2 white,3 red and 4 black balls.If X denotes the number of white balls drawn and Y denotes the number of red balls find the joint probability distribution of(X,Y).

5.Let X and Y have the joint probability function

y\x	0	1	2
0	0.2	0.3	0.15
1	0.1	0.1	0.25

i)Find $P(X + Y > 1)$. Ii) Are X and Y are independent random variables? Justify your answer.

6.The joint PDF of X and Y is given by $f(x, y) = \begin{cases} e^{-(x+y)}, & x \ge 0,\ y \ge 0 \\ 0, & otherwise \end{cases}$

Find i) the marginal density functions ii) are X and Y are independent. iii) P(X>Y) , P(X<1), P(X+Y<1)

iv) $P(X < 1 \cap Y < 3)$

7.If the joint pdf X and Y is given by $f(x,y) = \begin{cases} \frac{1}{8}(6-x-y), 0 < x < 2,\ 2 < y < 4. \\ 0, otherwise \end{cases}$

Find i) $P(X < 1 \cap Y < 3)$ ii) P(X+Y<3) iii) P(X<1 / Y<3) iv) f (y/x=2) v)P(X<1,Y<3).

8.Given $f(x,y) = \begin{cases} cx(x-y), 0 < x < 2, |y| < x \\ 0, otherwise \end{cases}$. Find i)c ii)f(x), f(y). iii) Conditional density function of y for x.

9.The joint PDF of random variable X and Y is given by $f(x,y) = \begin{cases} cxy^2, 0 \le x \le y \le 1 \\ 0, otherwise \end{cases}$

i)Find the value of C. ii)Find the marginal density functions of X and Y. iii) Find the conditional density functions $f_{Y/X}\left(Y/X\right)$ *and* $f_{X/Y}\left(X/Y\right)$.

10. The joint PDF of random variable X and Y is given by $f(x,y) = k\,xy\,e^{-(x^2+y^2)},\ x > 0,\ y > 0.$ Find the value of k. Prove that X and Y are independent.

11. The joint pdf of a two dimensional random variable (X,Y) is given by

$f(x,y) = xy^2 + \frac{x^2}{8},\ 0 \le x \le 2, 0 \le y \le 1.$ Compute $i) P(X > 1),\ P\left(Y < \frac{1}{2}\right),\ P(X < Y)$

$ii) P\left(X > 1 / Y < \frac{1}{2}\right),\ P\left(Y < \frac{1}{2} / X > 1\right)\ \ iii) P(X + Y \le 1).$

12. The joint PDF of the random variables X and Y is given by $f(x,y) = \begin{cases} 8xy, 0 < x < 1, 0 < y < x \\ 0, otherwise \end{cases}$

Find i) $f_X(x), f_Y(Y)\ \ ii) f(y/x),\ f(x/y)\ \ iii) P(X < 1/2)$.

13. Given the joint PDF f(x,y) as $f(x,y) = \begin{cases} \frac{1}{4}(1+xy), |x| \le 1, |y| \le 1 \\ 0, otherwise \end{cases}$. Find the marginal and conditional PDF of X and Y. Show that X and Y are independent.

14. If the joint distribution function of X and Y is given by

$F(x,y) = \left(1-e^{-x}\right)\left(1-e^{-y}\right),\ for\ x > 0,\ y > 0$

$= 0, otherwise.$

Find i) the marginal density of X and Y. ii) Are X and Y are independent. iii) The conditional density function of X and y iv) P(X<1), P(1<x<3, 1<y<2) v) $P(X \le 1 \cap Y \le 1)\ (or)\ P(X \le 1, Y \le 1)$ vi) $P(X + Y \ge 1)$.

15. i)Calculate the correlation coefficient for the following heights(in inches)of fathers X and their sons Y.

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

ii)Find the coefficient of correlation between industrial production X and export Y from the following data.

Production X	55	56	58	59	60	60	62
Export Y	35	38	37	39	44	43	44

iii) Calculate the correlation coefficient for the following data

X	9	8	7	6	5	4	3	2	1
Y	15	16	14	13	11	12	10	8	9

16. Two random variables X and Y have the following joint PDF

$$f(x,y)=\begin{cases}2-x-y, & 0\le x\le 1,\ 0\le y\le 1\\ 0, & otherwise\end{cases}$$ Find var (X), Var (Y) Cov(X,Y) and ρ_{XY}.

17. The joint p.d.f of R.Vs X and Y is given by $f(x,y)=\begin{cases}3(x+y), & 0\le x\le 1,\ 0\le y\le 1,\ x+y\le 1\\ 0, & otherwise\end{cases}$

Find the i)Marginal PDF os X, ii) P(X+Y<1/2), iii) Cov(X,Y)

18. If $f(x,y)=\frac{1}{8}(6-x-y),\ 0\le x\le 2,\ 2\le y\le 4.$ Find the correlation coefficient between X and Y.

19. Determine the correlation coefficient between R.Vs X and Y whose j.p.d.f is

$$f(x,y)=\begin{cases}x+y, & 0\le x\le 1,\ 0\le y\le 1\\ 0, & otherwise\end{cases}$$

20. From the following data find i) the two regression equations. ii) the coefficient of correlation between the marks in economics and statistics. iii) the mosy likely marks in statistics when marks in economics are 30.

Marks in Economics	25	28	35	32	31	36	29	38	34	32
Marks in Statistics	43	46	49	41	36	32	31	30	33	39

21. From the following data obtain two regression equations

X	6	2	10	4	8
Y	9	11	5	8	7

22. The two lines of regression are 8x-10y+66=0 and 40x-18y-214=0. The variance of X is 9. Find
i)the mean values of X and Y. ii)the correlation coefficient between X and Y.
iii) the standard deviation of Y.

23. If the joint PDF of a two dimensional random variable (X,Y) is given by
$f(x,y)=x+y,\ 0\le x\le 1,\ 0\le y\le 1.$ Find the PDF of U=XY.

24. If X and Y each follow exponential distributions with parameter 1 and are independent. Find the

PDF of U=X - Y.

25. The joint PDF of X and Y are given by $f(x,y)=e^{-(x+y)},\ x>0,\ y>0$. Find the PDF of $U=\dfrac{X+Y}{2}$.

26. Let X and Y be positive independent random variable with the identical probability density function

$f(x)=e^{-x},\ x>0.$ Find the joint probability density function of U=X+Y and V=X/Y. Are U and V independent?.

27. If X and y are independent random variables with density function $f_X(x)=\begin{cases}1, 1\le x\le 2\\ 0, otherwise\end{cases}$ and $f_Y(y)=\begin{cases}\frac{y}{6}, 2\le y\le 4\end{cases}$, find the density function of ***Z=XY***.

28. ..If X and Y are independent RVs each normally distributed with mean zero and variance σ^2 find the pdf of $R=\sqrt{X^2+Y^2}$ $and\ \emptyset = tan^{-1}\left(\frac{Y}{X}\right)$

UNIT III

CLASSIFICATION OF RANDOM PROCESSES

1. Explain the classification of random variables
2. Explain the classification of Markov process.
3. Obtain the steady state or long run probabilities for the population size of a birth death process. (or) Discuss the pure birth process and hence obtain its probabilities $P_{n(t)},\ n=1,2,3,\ldots$ mean and variance.
4. The probability distribution of the process $\{X(t)\}$ is given by

$$P(X(t)=n)=\begin{cases}\dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n=1,2,3,\ldots\\[2mm] \dfrac{(at)}{(1+at)}, & n=0.\end{cases}$$

. Show that $\{X(t)\}$ is not stationary.

5. Show that the random process $X(t)=A\sin(\omega t+\theta)$ is wide sense stationary. If A and ω are constants and θ is uniformly distributed random variable in $(0,2\pi)$.
6. Show that the random process $X(t)=A\cos(\omega t+\theta)$ where A and ω are constants and θ is uniformly distributed in $(-\pi,\pi)$. is wide sense stationary.
7. Two random process X(t) and Y(t) are defined by $X(t)=A\cos\omega t+B\sin\omega t$ and $Y(t)=B\cos\omega t-A\sin\omega t$. Show that X(t) and Y(t) are jointly wide sense stationary, if A and B are uncorrelated random variables with zero means and the same variances and ω is a constant.
8. Show that $X(t)=A\cos\lambda t+B\sin\lambda t$ (where A and B are two random variables) is wide sense stationary if i) E(A)= E(B)=0, ii) $E(A^2)= E(B^2)=0$ and E(AB)=0.

9. A fair die is tossed repeatedly. If X_n denotes the maximum of the numbers occurring in the first 'n' tosses , find the transition probability matrix P of the Markov Chain $\left\{X_{n,n\geq 0}\right\}$. Find also P^2 and $P\{X_2=6\}$.

10. The one step TPM of Markov chain $\{X_n\}$ n=1,2,3,… having three states 1,2,3 and

$$p^{(1)}=\begin{bmatrix}0.1 & 0.5 & 0.4\\ 0.6 & 0.2 & 0.2\\ 0.3 & 0.4 & 0.3\end{bmatrix}$$ and the initial distribution is $p^{(0)}=[0.7 \quad 0.2 \quad 0.1]$.

Find $i)P(X_2=3).\ ii)P\left(X_2=3/X_0=1\right)iii)P(X_3=2,X_2=3,X_1=3,X_0=2)$

11. Find the nature of the states of the Markov chain with the transition probability matrix $\begin{bmatrix}0 & 1 & 0\\ \frac{1}{2} & 0 & \frac{1}{2}\\ 0 & 1 & 0\end{bmatrix}$

12. The TPM of Markov chain with three states (0,1,2) is $p=\begin{bmatrix}3/4 & 1/4 & 0\\ 1/4 & 2/4 & 1/4\\ 0 & 3/4 & 1/4\end{bmatrix}$ and the initial distribution of the chain is $P(X_0=i)=1/3$, $i=0,1,2.$.

Find $i)P(X_2=2).\ ii)P\left(X_2=1,X_0=0\right)iii)P(X_3=1,X_2=2,X_1=1,X_0=2)$ iv) Is the chain is irreducible? Explain

13. Consider the Markov chain $\{X_n, n\geq 0\}$ with state space $S=\{0,1\}$ and one step TPM $P=\begin{bmatrix}1 & 0\\ \frac{1}{2} & \frac{1}{2}\end{bmatrix}$ .i)Draw the state transition diagram. ii)Is the chain irreducible? Explain. Iii)Show that state 1 is transient.

14. Three boys A,B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the transition matrix and classify the states.

15. Find the mean and autocorrelation of Poisson process.

16. A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work iff a 6 appeared. Find i) the probability that he takes a train on the 3rd day. ii) the probability that he drives to work in the long run.

17. Prove that the sum of two independent poisson process is again a poisson process. (or) State and prove the additive property of poisson process.

18. Prove that the difference of two independent poisson process is not a poisson process.

19. Prove that the poisson process is a Markov process.

20. Suppose that the customer arrive at a bank according to a poisson process with a mean rate of 3 per minute. Find the probability that during a time interval of (0,2) min i) Exactly 4 customers.ii) more than 4 customers arrive.

21. A gambler has Rs.2.He bets Re.1 at a time and wins Re.1 with probability ½ . He stops playing if he loses Rs.2 or wins Rs.4. (i) What is the tpm of the related Markov chain ? (ii) What is the

probability that he has lost his money at the end of 5 plays?

UNIT: IV CORRELATION AND SPECTRAL DENSITIES

PROBLEMS BASED ON AUTO CORRELATION FUNCTION

1.Find the auto correlation function of the periodic time function of the period time function $\{X(t)\} = A\sin \omega t.$

2.If {X(t)} and {Y(t)}are two random processes with auto correlation function $R_{XX}(\tau)$ and $R_{YY}(\tau)$ respectively then prove that $|R_{XY}(\tau)| \leq \sqrt{R_{XX}(0)\, R_{YY}(0)}$ .Establish any two properties of auto correlation function $R_{XX}(\tau)$.(2)

3.A stationary random process X(t) with mean 2 has the auto correlation function $R_{XX}(\tau) = 4 + e^{\frac{-|\tau|}{10}}$ .Find the mean and variance of $Y = \int_0^1 X(t)\, dt$.

PROBLEMS BASED ON CROSS CORRELATION FUNCTION

4.Two random process X(t) and Y(t) are defined as follows :
$X(t) = A\cos(\omega t + \theta)$ *and* $Y(t) = B\sin(\omega t + \theta)$ where A,B and ω are constants ; θ is a uniform random variable over (0,2 π).Find the cross – correlation function of X(t) and Y(t).

PROBLEMS BASED ON CROSS CORRELATION & AUTO CORRELATION FUNCTION

5. {X(t)} and {Y(t)} are zero mean and stochastically independent random processes having auto correlation functions $R_{XX}(\tau) = e^{-|\tau|}$ *and* $R_{YY}(\tau) = \cos 2\pi\tau$ respectively.Find
(i) The auto correlation function of W(t)=X(t)+Y(t) and Z(t)=X(t)-Y(t)
(ii)The cross correlation function of W(t) and Z(t).

PROBLEMS BASED ON POWER SPECTRAL DENSITY

6.A Random Process $\{X(t)\}$ is given by $X(t) = A\cos pt + B\sin pt$, where A and B are independent random variables such that $E(A) = E(B) = 0$ and $E(A^2) = E(B^2) = \sigma^2$ .Find the power spectral density of the process.

AUTO CORRELATION FUNCTION TO POWER SPECTRAL DENSITY

7.The auto correlation function of the random binary transmission {X(t)} is given by $R(\tau) = 1 - \frac{|\tau|}{T}$ *for* $|\tau| < T$ *and* $R(\tau) = 0$ *for* $|\tau| < T$ .Find the power spectrum of the process {X(t)}.

8. Find the power spectral density of the random process whose auto correlation function is
$$R(\tau) = \begin{cases} 1 - |\tau|, \; for |\tau| \leq 1 \\ 0 \qquad , otherwise \end{cases}$$
(2)

9. The auto correlation function of a WSS process is given by $R(\tau) = \alpha^2 e^{-2\lambda|\tau|}$ determine the power spectral density of the process.

10.Find the power spectral density of WSS process with autocorrelation function
$$R(\tau) = e^{-\alpha\tau^2}$$

11.The auto correlation function of a random process is given by $R(\tau) = \begin{cases} \lambda^2 ; & |\tau| > \varepsilon \\ \lambda^2 + \frac{\lambda}{\varepsilon}\left(1 - \frac{|\tau|}{\varepsilon}\right); & |\tau| \leq \varepsilon \end{cases}$

Find the power spectral density of the process.(2)

12.Given the power spectral density of a continuous process as $S_{XX}(\omega)=\frac{\omega^2+9}{\omega^4+5\omega^2+4}$.Find the mean square value of the process.

13.Find the power spectral density function whose auto correlation function is given by $R_{XX}(\tau)=\frac{A^2}{2}\cos(\omega_0\tau)$.

14.Let X(t) and Y(t) be both zero-mean and WSS random processes consider the random process Z(t) defined by Z(t)=X(t)+Y(t).Find

(i) The auto correlation function and the power spectrum of Z(t) if X(t) and Y(t) are jointly WSS.

(ii) The power spectrum of Z(t) if X(t) and Y(t) are orthogonal.

15. Define spectral density of a stationary random process X(t).Prove that for a real random process X(t) the power spectral density is an even function

16.The auto correlation function of the random telegraph signal process is given by $R(\tau)=\alpha^2 e^{-2\sqrt{|\tau|}}$ determine the power density spectrum of the random telegraph signal process.

17.If the process $\{X(t)\}$ is defined as X(t)=Y(t)Z(t) where $\{Y(t)\}$ and $\{Z(t)\}$ are independent WSS processes, prove that

(1) $R_{XX}(\tau)=R_{YY}(\tau)R_{ZZ}(\tau)$ and

(2) $S_{XX}(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}S_{YY}(\alpha)S_{ZZ}(\omega-\alpha)d\alpha$.

18.Prove that the random processes $X(t)$ and $Y(t)$ defined by $X(t)=A\cos\omega t+B\sin\omega t$ and $Y(t)=B\cos\omega t-A\sin\omega t$ are jointly WSS processes.

POWER SPECTRAL DENSITY FUNCTION TO AUTO CORRELATION FUNCTION

19. Find the auto correlation function of the process {X(t)} for which the power spectral density is given by $S_{XX}(\omega)=1+\omega^2 \; for\,|\omega|<1 \; and \; S_{XX}(\omega)=0 \; for\,|\omega|>1$.

20.The power spectral density function of a zero mean WSS process X(t) is given by

$S(\omega)=\begin{cases}1, & |\omega|<\omega_0\\ 0, & otherwise\end{cases}$ .Find $R(\tau)$ and show that X(t) and $X\left(t+\frac{\pi}{\omega_0}\right)$ are uncorrelated.

21. If the power spectral density of a WSS process is given by $S(\omega)=\begin{cases}\frac{b}{a}(a-|\omega|), & |\omega|\le a\\ 0, & |\omega>a|\end{cases}$

Find the autocorrelation function of the process.

CROSS CORRELATION FUNCTION TO CROSS POWER SPECTRAL DENSITY

22.The cross – correlation function of two processes X(t) and Y(t) is given by

$R_{XY}(t,t+\tau)=\frac{AB}{2}\{\sin(\omega_0\tau)+\cos\omega_0[(2t+\tau)]\}$ where A,B and ω_0 are constants.

Find the crosspower spectrum $S_{XY}(\omega)$.

CROSS POWER SPECTRAL DENSITY FUNCTION TO CROSS CORRELATION

23.The cross power spectram of real random processes {X(t)} and {Y(t)} is given by

$S_{XY}(\omega)=\begin{cases}a+bj\omega, & for|\omega|<1\\ 0, & elsewhere\end{cases}$ .Find the cross correlation function.(3)

24. If the cross power spectral density of X(t) and Y(t) is $S_{XY}(\omega)=\begin{cases}\alpha+\frac{ib\omega}{\alpha}\ ;\ -\alpha<\omega<\alpha,\alpha>0\\ 0\ ;\ otherwise\end{cases}$

where α and b are constants. Find the cross correlation function .

UNIT-V LINEAR SYSTEMS WITH RANDOM INPUTS

PROBLEMS BASED ON LINEAR INVARINT SYSTEM

1. Let X(t) be WSS process which is the input to a linear time invariant system with unit impulse h(t) and output Y(t) , then prove that $S_{YY}(\omega)=|H(\omega)|^2 S_{XX}(\omega)$.

2. If {X(t)} is a WSS process and if $Y(t)=\int_{-\infty}^{\infty} h(\xi)X(t-\xi)d\xi$ then prove that

(i). $R_{XY}(\tau)=R_{XX}(\tau)*h(\tau)$ where * stands for convolution.

(ii). $S_{XY}(\omega)=S_{XX}(\omega)*H(\omega)$.

3. If the input to a time invariant, stable, linear system is a WSS process ,prove that the output will also be a WSS process.

4. .Find the system transfer function ,if a Linear Time Invariant system has an impulse function

$$H(t)=\begin{cases}\frac{1}{2c};|t|\le c\\ 0;|t|\ge c\end{cases}$$

PROBLEMS UNDER AUTO CORRELATION & CROSS CORRELATION OF INPUT AND OUTPUT

5.A wide sense stationary random process $\{X(t)\}$ with autocorrelation $R_{XX}(\tau)=e^{-a|\tau|}$ Where A and a are real positive constants,is applied to the input of an linear transmission input system with impulse response $h(t)=e^{-bt}u(t)$ where b is a real positive constant. Find the autocorrelation of the output Y(t) of the system.

6.Assume a random process X(t) is given bas an input to a system with transfer function $H(\omega)=1\, for -\omega_0<\omega<\omega_{0.}$ If the autocorrelation function of the input process is $\frac{N_0}{2}\delta(t)$, find the autocorrelation function of the output process.

7. A system has an impulse response $h(t)=e^{-\beta t}U(t)$,find the power spectral density of the Show that if the input $\{X(t)\}$ is a WSS process for a linear system then output {Y(t)} is a WSS process.Also find $R_{Xy}(\tau)$.output Y(t) corresponding to the input X(t).

8.If X(t) is the input voltage to a circuit and Y(t) is the output voltage.{X(t)} is a stationary random process with μ_X *and* $R_{XX}(\tau)=e^{-2|\tau|}$.Find the mean μ_Y and power spectrum $S_{YY}(\omega)$ of the output if the system transfer function is given by $H(\omega)=\frac{1}{\omega+2i}$.

9.Consider a system with transfer function $\frac{1}{1+j\omega}$.An input signal with autocorrelation function $m\delta(\tau)+m^2$ is fed as input to the system. Find the mean and mean-square value of the output.(2)

10.A stationary random process X(t) having the autocorrelation function $R_{XX}(\tau)=A\delta(\tau)$ is applied to a linear system at time t=0 where $f(\tau)$ represents the impulse function. The linear system has the impulse response of $h(t)=e^{-bt}u(t)$ where u(t) represents the impulse function. Find $R_{YY}(\tau)$. Also find the mean and variance of Y(t).(2)

11.A linear system is described by the impulse response $h(t)=\frac{1}{RC}e^{\frac{-t}{RC}}u(t)$. Assume an input process whose autocorrelation function is $B\delta(t)$. Find the mean and Autocorrelation function of the output process.

12.For a input –output linear system (X(t),h(t),Y(t)),drive the cross correlation function $R_{XY}(\tau)$ and the output autocorrelation function $R_{YY}(\tau)$.

13.Prove that the spectral density of any WSS process is non-negative .

14.X(t) is the input voltage to a circuit (system) and Y(t) is the output voltage.{X(t)} is a stationary random process with $\mu_x = 0$ and $R_{XX}(\tau) = e^{-\alpha|\tau|}$. Find $\mu_y, S_{yy}(\omega)$ *and* $R_{YY}(\tau)$,if the power transfer function is $H(\omega) = \dfrac{R}{R + iL\omega}$. $Y(t) = \int_{-\infty}^{\infty} h(\alpha) X(t-\alpha) d\alpha$.

15.A random process X(t) is the input to a linear system whose impulse function is $h(t) = 2e^{-2t}; t \geq 0$.The autocorrelation function of the process is $R_{XX}(\tau) = e^{-2|\tau|}$.Find the power spectral density of the output process Y(t).

16.Check whether the following systems are linear(i) y(t) = t x(t) (ii) y(t) = x^2(t).

17.The power spectral density of a signal x(t) is $S_x(\omega)$ and its power is P. Find the power of the signal bx(t).

UNIT IV - QUEUING THEORY

1. **What do you mean by transient state and steady state in queuing system?(3)**

   Solution: Transient state: A system is said to be transient state then its operating characteristics are dependent of time.

   Steady state: A system is said to be steady state when the behavior of the system is independent of time.

2. **Define Kendall's notation of queuing model.**

   Solution: The assumptions made for the simplest queuing model is (a/b/c):(d/e) where

   a=Probability distribution of the inter arrival time

   b= Probability distribution of the service time

   c=Number of servers in the system

   d=Maximum number of customers allowed in the system

   e=Queue discipline.

3. **What are the basic characteristics of Queuing process? (or) State the characteristic of the queuing theory.**

   Solution: i) Arrival (or) Input pattern of the customers

   ii) Service Pattern (or) Service Mechanism of server

   iii) Queue Discipline iv) System Capacity.

4. **State the various disciplines in queuing model.(2)**

   Solution: i) FIFO-First In First Out (or) FCFS-First Come First Serve

   ii) LIFO-Last In First Out (or) LCFS-Last Come First Serve

   iii) SIRO-Service (Selection) In Random Order iv)GD-General Service Discipline.

5. **Explain your understanding of the relationship between the arrival rate λ and inter arrival time.**

   Solution: Arrival Rate: The number of customers arriving per unit of time.

   Inter Arrival Rate: The time between two successive arrivals.

6. **Define various distributions in arrival at service mechanism.**

   Solution: For Arrival –Poisson distribution, For service-Exponential distribution.

7. **For (M/M/1):(∞ /FIFO), write down the Little's formula. (or) Write the Little's formula for the infinite capacity queueing models. (or) Define Little's formula.(4)**

Solution: $i) L_s = \frac{\lambda}{\mu - \lambda} = \lambda W_s \quad ii) L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \lambda W_q \quad iii)\ W_s = W_q + \frac{1}{\mu} \quad iv) W_q = \frac{L_q}{\lambda}$.

8. **What is the total queue length in M/M/1?**

Solution: $L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$.

9. **Give the formula for average number of customers in the system in single server models..**

Solution: For $(M/M/1):(\infty/FIFO)$ Model $L_s = \frac{\lambda}{\mu-\lambda}$

For $(M/M/1):(K/FIFO)$ Model $L_s = \frac{1}{C!C}\left(\frac{\lambda}{\mu}\right)^{C+1} \frac{1}{1-\left(\frac{\lambda}{\mu}\right)^2} P_0 + \frac{\lambda}{\mu}$

10. **For (M/M/C):(N /FIFO) model, write down the formula for**

a) average number of customers in the queue.

b) average waiting time in the system.

Solution: a) Average number of customers in the queue: L_q

$$L_q = P_0\left(\frac{\lambda}{\mu}\right)^C \frac{\left(\frac{\lambda}{C\mu}\right)}{C!\left(1-\left(\frac{\lambda}{C\mu}\right)\right)}\left[1-\left(\frac{\lambda}{C\mu}\right)^{K-C} - (K-C)\left(1-\frac{\lambda}{C\mu}\right)\left(\frac{\lambda}{C\mu}\right)^{K-C}\right]$$

b) Average waiting time in the system W_s: $W_s = \frac{L_s}{\lambda'}$ **.**

11. **In the usual notation of M/M/1 queuing system $\lambda = 12$ per hour and $\mu = 24$ per hour. Find the average number of customers in the system.(2)**

Solution: Given $\lambda = 12$ **/ hr,** $\mu = 24$ **/ hr**

To find the average number of customers in the system:L_S

$$L_s = \frac{\lambda}{\mu-\lambda} = \frac{12}{24-12} = \frac{12}{12} = 1\ Customer$$

12. **What is the probability that a customer has to wait more than 15 mins to get his service completed in a M/M/1 queuing system, if $\lambda = 6/hr$ hrs and $\mu = 10/hr$.(3)**

Solution: Given $\lambda = 6/hr$ **hr,** $\mu = 10/hr$

To find P [a customer has to wait more than 15 mins]:

We know that $P(W_S > t) = e^{-(\mu-\lambda)t}$

$$P(W_S > 15\,\text{min}) = P\left(W_S > 15 \times \frac{1}{60} hr\right) = e^{-(10-6)\frac{1}{4}} = e^{-1} = 0.367$$

13. **What is the probability that an arrival to an infinite capacity 3 server Poisson queuing system with $\frac{\lambda}{\mu} = 2$ *and* $P_0 = \frac{1}{9}$ enters the service without waiting?**

Solution: P [without waiting]= P(N<3)

$$= P_0 + P_1 + P_2 \qquad \left[\because P_n = \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0\right]$$

$$\therefore P(N < 3) = \frac{1}{9} + \frac{2}{9} + \frac{2}{9} = \frac{5}{9}\ .$$

14. **If there are two servers in an infinite capacity Poisson queue system with $\lambda = 10$ per hour and $\mu = 15$ per hour. What is the percentage of idle time for each server?**

Solution: Given $\lambda = 10$**,** $\mu = 15$ and c=2.

To find P [Server is idle]:P_0

$$P_0 = 1 - \frac{\lambda}{c\mu} = 1 - \frac{10}{2 \times 15} = \frac{2}{3}$$

$$\%\text{ of idle time} = \frac{2}{3} \times 100 = 66.66\%.$$

15. **Given $\lambda = 10$ hrs and $\mu = {}^{1}\!/_{3}$ mins. Find ρ.**

Solution: Given $\lambda = 10$**/hr,** $\mu = \frac{1}{3} = \frac{1}{3} \times 60 = 20/hr$ **.** $\rho = \frac{\lambda}{\mu} = \frac{10}{20} = \frac{1}{2}$

16. **In a given M/M/1/FCFS queue, ρ=0.7. Find the expected number of customers in the queue.**

Solution: Given $\rho = 0.7 \Rightarrow \frac{\lambda}{\mu} = 0.7$

To Find the expected number of customers in the queue: L_q

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = L_s - \rho$$

$$WKT\ \ L_s = \frac{\rho}{1-\rho} = \frac{0.7}{1-0.7} = 2.3$$

$$\therefore L_q = 2.3 - 0.7 = 1.6$$

17. **Consider an M/M/C queuing system. Find the probability that an arriving customer is forced to join the queue.**

Solution: The probability that an arrival is forced to join in the queue

= the probability that an arrival has to wait

=probability that there are C or more customers in the system = $P(N \geq C)$

$$\therefore P(N \ge C) = \sum_{n=C}^{\infty} P_n = \sum_{n=C}^{\infty} \frac{1}{C!C^{n-c}} \left(\frac{\lambda}{\mu}\right)^n P_0$$

$$= \frac{1}{C!}\left(\frac{\lambda}{\mu}\right)^C P_0 \sum_{n=C}^{\infty}\left(\frac{\lambda}{C\mu}\right)^{n-C} = \frac{1}{C!}\left(\frac{\lambda}{\mu}\right)^C P_0\left[1+\frac{\lambda}{C\mu}+\left(\frac{\lambda}{C\mu}\right)^2+\ldots\right]$$

$$= \frac{1}{C!}\left(\frac{\lambda}{\mu}\right)^C P_0\left[1-\frac{\lambda}{C\mu}\right]^{-1}$$

18. **In a given M/M/1 queue, the arrival rate $\lambda = 7$ customers / hour and service rate $\mu = 10$ customers / hour. Find $P(X \ge 5)$, where X is the number of customers in the system.**

Solution: Given $\lambda = 7/hr$, $\mu = 10/hr$

To Find $P(X \ge 5)$: ie) $P(N \ge 5)$

We know that $P(N \ge n) = \left(\frac{\lambda}{\mu}\right)^n \Rightarrow P(N \ge 5) = \left(\frac{7}{10}\right)^5 = 0.1681$.

19. **In a 3 server infinite capacity Poisson queue model if $\frac{\lambda}{C\mu} = \frac{2}{3}$. Find P_0.**

Solution: Given $\frac{\lambda}{C\mu} = \frac{2}{3}$, $C = 3. \Rightarrow \frac{\lambda}{\mu} = 2$

$$P_0 = \frac{1}{\sum_{n=0}^{C-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \left(\frac{\lambda}{\mu}\right)^C \frac{1}{C!\left(1-\frac{\lambda}{C\mu}\right)}} = \frac{1}{\sum_{n=0}^{2}\frac{1}{n!}(2)^n + (2)^3 \frac{1}{3!\left(1-\frac{2}{3}\right)}}$$

$$= \frac{1}{1+2+2+\left(\frac{8\times 3}{6\times 1}\right)} = \frac{1}{9}.$$

20. **If $\lambda = 6/hr$, $\mu = 4/hr$ and N=2, find the probability for no customers in a single server , finite capacity model.**

Solution: Given $\lambda = 6/hr$, $\mu = 4/hr$ and N=2.

To find P [No customers in the system]: P_0

$$P_0 = \frac{1-\left(\frac{\lambda}{\mu}\right)}{1-\left(\frac{\lambda}{\mu}\right)^{N+1}} = \frac{1-\left(\frac{6}{4}\right)}{1-\left(\frac{6}{4}\right)^{2+1}} = \frac{-4}{8-27} = \frac{4}{19} = 0.21.$$

UNIT V - NON MARKOVIAN QUEUES AND QUEUING NETWORKS

1. **State Pollaczek – Khintchine formula and explain the notations.(or) Write Pollaczek – Khintchine formula for the case when service time distribution is Erlang distribution with K phases. (or) In (M/G/1) model, what is the formula for the average number of customers in the system? (4)**

Solution: $L_s = \lambda E(T) + \dfrac{\lambda^2\left(V(T)+E^2(T)\right)}{2(1-\lambda E(T))}$

where λ =Mean arrival rate, E(T)=Mean of T, V(T)=Variance of T.

2. **What do you mean by M/G/1 queue and define the Little's formula for M/G/1 queue model.**

   Solution: It is a Non-Markovian model where M indicates the number of arrivals in time t which follows a Poisson process. **Little's formula is**

   i) $L_s = \lambda E(T) + \dfrac{\lambda^2\left(V(T)+E^2(T)\right)}{2(1-\lambda E(T))}$ ii) $L_q = L_s - \dfrac{\lambda}{\mu}$ $iii)\ W_s = \dfrac{L_s}{\mu}$ $iv) W_q = \dfrac{L_q}{\lambda}$

3. **Define effective arrival rate with respect to an (M/M/1) : (G_D/ N / ∞)queuing model. (or) Explain effective arrival rate in a finite queue capacity model.(or) What is the effective arrival rate for M/M/1/N queuing system.**

   Solution: Effective arrival rate $\lambda' = \mu(1-P_0)$.

4. **Write classification of Queuing Networks.**

   Solution: i) Open Queuing Networks and ii) Closed Queuing Networks

5. **Define closed queuing network or closed Jackson network.**

   Solution: Cases for which r_i =0 (no customers may enter the system from outside) and r_{i0} (no customers may leave the system) for all i .

6. **Define an open network or open Jackson Network.**

   Solution: An open Jackson network is a system of 'k' service stations were station 'i' has the following characteristics

   i)An infinite queue capacity

   ii) Customer arrive at station 'i' from outside the system according to poisson process with parameter a_i.

   iii) c_i servers at station I with an exponential service time distribution with parameter μ_i

   iv) Customers completing service at station i next go to station j with probability P_{ij}

7. **State arrival theorem.**

   Solution: In a closed network system with N customers, the system as seen by arrivals to server j is distributed as the stationary distribution in the same network system, where there are only N-1customers.

8. **Explain series queue model.**

   Solution: In this model we have a series of service stations through which each calling unit must progress prior to leaving the system.

   Ex: A registration process for the admission into an university where the applicant must visit the series of desks such as advisor, cashier, department etc.

9. **Define a two – stage series queue.**

   Solution: Consider a two stage queuing system in which customers arrive from outside at a Poisson rate λ to S_1. After being served at S_1 then they join the queue in front of S_2. After getting the service at S_2

then they leave the system. Each server serves one customer at a time and the service times follows distributions with parameter μ_1 *and* μ_2 respectively.

10. **A one man barber shop takes exactly 25 minutes to complete one haircut. If customers arrive at the barber shop in a poisson fashion at an average rate of one every 40 minutes, how long on the average a customer spends in the shop.**

Solution: Since the service time is constant =25 min. $\therefore E(T) = 25$ and V(T)=0.

To find the average a customer spends in the shop W_s : $W_s = \frac{L_s}{\lambda}$

We know that by P-K formula

$$L_s = \lambda E(T) + \frac{\lambda^2\left(V(T) + E^2(T)\right)}{2(1-\lambda E(T))} = \frac{25}{40} + \frac{\left(\frac{25}{40}\right)^2\left[0+25^2\right]}{2\left(1-\frac{25}{40}\right)} = \frac{25}{40} + \frac{625}{1000} \times \frac{4}{3} = \frac{55}{48}$$

$$\therefore W_s = \frac{L_s}{\lambda} = \frac{\frac{55}{48}}{\frac{1}{40}} = 45.8\,\text{min}$$

11. **If people arrive to purchase cinema tickets at the average rate of 6 per minutes, it takes an average of 7.5 seconds to purchase a ticket. If a person arrives 2 mins before the picture starts and it takes exactly 1.5min to reach the correct seat after purchasing the ticket. Can he expect to be seated for the start of the picture.**

Solution: Given $\lambda = 6, \frac{1}{\mu} = 7.5 = \frac{15}{2} \Rightarrow \mu = \frac{2}{15} \times 60 = 8$

To find he expect to be seated for the start of the picture ie) W_s : $W_s = \frac{1}{\mu - \lambda} = \frac{1}{2}$.

12. **Consider a service facility with two sequential stations with respective service rates 3/min and 4/min. The arrival rate is 2/min. What is the average service time of the system of the system , if the system could be approximated by a two stage tandem queue?**

Solution: Given $\lambda = 2/\text{min}$. $\mu_1 = 3/\text{min}$, $\mu_2 = 4/\text{min}$

Average service time = W_s at station I + W_s at station II

$$= \frac{1}{\mu_1 - \lambda} + \frac{1}{\mu_2 - \lambda} = \frac{1}{3-2} + \frac{1}{4-2} = \frac{3}{2}$$

13. **In an M/G/1/FCFS with infinite capacity queue, the arrival rate** $\lambda = 5$ **and the mean service time** $E(S) = \frac{1}{8}$ **hour and Var(S)=0. Compute the mean waiting time W_q in the queue.**

[Hint E[T]=1/8, V[T]=0].

14. **In an M/G/1 model if** λ =5min , $\mu = 6\,\text{min}$ *and* $\sigma = \frac{1}{20}$. **Find the value of L_s.**

PART - B

UNIT I RANDOM VARIABLES

10. A random variable X has the following probability function

Values of X	0	1	2	3	4	5	6	7	8
P(x)	a	3a	5a	7a	9a	11a	13a	15a	17a

i)Determine the value of a. ii) Find $p(X<3),\ p(X\geq 3),\ p(0<X<5).$

iii) Find the distribution function of X.

11. A random variable X ha the following probability function

X	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	K^2	$2K^2$	$7K^2$+k

i)Determine the value of k. ii) Find $p(X<6),\ p(X\geq 6),\ p(0<X<5).$

iii) Find $p(1.5<X<4.5/X>2)$ iv)Find the distribution function of X.

12. The density function of continuous random variable X is given by

$$f(x)=\begin{cases} ax\ ,\ 0\leq x<1, \\ a\ ,\ \ 1\leq x<2, \\ 3a-ax\ ,\ 2\leq x<3, \\ 0\ ,\ \ otherwise. \end{cases}$$

i) Find the value of a. ii) Cumulative distribution of X.

13. A cumulative distribution function is given by $F(x)=\begin{cases} 0, & x<0, \\ x^2, & 0\leq x<\frac{1}{2}, \\ 1-\frac{3}{25}(3-x^2), & \frac{1}{2}\leq x<3, \\ 1, & x\geq 3. \end{cases}$

Find i) PDF of X. ii) $p(\frac{1}{3}\leq x<4)$. iii) $p(|x|\leq 1)$.

14. Let X be a discrete RV whose cumulative distribution function is

$$F(x)=\begin{cases} 0, & for\ x<-3, \\ 1/6, & for\ -3\leq x<6, \\ 1/2, & for\ 6\leq x<10, \\ 1, & for\ x\geq 10. \end{cases}$$

Find i) PDF of X. ii) $P(X\leq 4),\ P(-5<X\leq 4)$.

15. A continuous random variable X has the PDF $f(x)=3x^2,\ 0<x<1$. Find a and b such that $i)P(X\leq a)=P(X>a).\ \ ii)P(X>b)=0.05$

16. A random variable X has the following probability function Find K and the mean of X.

X	-1	0	1	2
P(X=x)	0.4	k	0.2	0.3

17. A random variable X has the following probability function

X	-1	0	1	2
P(X=x)	1/3	1/6	1/6	1/3

Find

$i)\ E(X)\ ii) E(X^2)\ iii) E(2X+3)^2\ \ iv) Var(X)$

18. A continuous random variable X has the PDF $f(x) = Kx(2-x),\ 0 \le x \le 2$.Find i) K, ii) Mean and iii) Var (X).

10. The density function of continuous random variable X is given by $f(x) = \begin{cases} Ae^{-x}, & 0 < x < \infty, \\ 0, & otherwise. \end{cases}$

i) Find the value of A. ii) r^{th} moment about origin iii) 3^{rd} moment about mean.

11.Find MGF of $f(x) = \begin{cases} x, & 0 < x \le 1, \\ 2-x, & 1 \le x < 2, \\ 0, & otherwise. \end{cases}$ hence find mean and variance.

12.Let the random variable X have the p.d.f $f(x) = \begin{cases} \frac{1}{2}e^{-\frac{x}{2}}, & x > o \\ 0, & otherwise. \end{cases}$. Find the MGF, Mean and Variance of X.

13.A continuous random variable X has the PDF $f(x) = \begin{cases} Cxe^{-x}, & x > o \\ 0, & x \le 0. \end{cases}$ Find the value of C. Obtain the MGF, mean and variance.

14.A continuous random variable X has the PDF $f(x) = Kx^2e^{-x},\ x \ge 0.$ Find the r^{th} moment of X about the origin. Hence find the first four moments about mean, mean and variance.

15.A random variable X has the probability function $P(x) = \frac{1}{2^x},\ x = 1, 2, 3, \ldots$ Find MGF, Mean and variance.

16.If the moments of a random variable X are defined by $E(X^r) = 0.6;\ r = 1, 2, 3, \ldots.$ show that $P(X=0) = 0.4, P(X=1) = 0.6,\ P(X \ge 2) = 0.$

17.The random variable X has the MGF $M_X(t) = \frac{2}{2-t}$, Find mean and variance ox X.

18. Derive the MGF, mean and variance of binomial distribution with parameters n and p.

19.The mgf of x is given by $M_X(t) = (0.6e^t + 0.4)^8$ find the variance of x and mgf of Y=3X+4.

20.In a large consignment of electric bulbs 10% are defective. The random sample of 20 is taken for inspection. Find the probability that i) All are good bulbs. ii) Atmost there are 3 defectives. iii) Exactly there are 3 defectives.

21.A machine manufacturing screws is known to produce 5% defective. In a random sample of 15 screws, what is the probability that there are i) Exactly 3 defectives. ii) Not more than 3 defectives.

22. Out of 800 families with four children each, How many family would be expected to have (i) two boys and two girls. ii) atleast one boy iii) atmost 2 g irls . Asssume that equal probability for boys and girls.

23. If X is a binomial distributed R.V with E(X)=2 and Var(X)=4 / 3, find P(X=5).

24. Derive the MGF, Mean and Variance of Poisson distribution with parameter λ.

25. Derive the probability mass function of Poisson distribution as a limiting case of Binomial

distribution.

26. Six coins are tossed 6400 times using Poisson distribution what is the approximate probability of getting 6 heads in 10 times.

27.A number of monthly breakdown of a computer is a random variable X having a Poisson distribution with mean 1.8. Find the probability that this computer will function for a month i) without breakdown ii) with only one breakdown iii) atleast one breakdown.

28. Find the MGF of Geometric random variable with pdf $P(X=x)=pq^{x-1}, x=1,2,3,...$ and hence obtain its mean and variance.

29.Establish the memory less property of geometric distribution

30.A die is cast until 6 appears. What is the probability that it must be cast more than 5 (or 4)times?

31.Derive the MGF, Mean and Variance of negative binomial distribution.

32.Suppose that X is a negative binomial random variable with p=0.2 and r=4. Determine the mean of X.

33.Derive the MGF , mean and variance of uniform distribution.

34. A random variable X has a uniform distribution over the interval $(-3,3)$. Find

$$i) P(X<2), ii) P(|X|<2), iii) P(|X-2|<2), iv) Find\ K\ such\ that\ P(X>K)=\frac{1}{3}.$$

35. If X is uniformly distributed over $(-\alpha,\alpha), \alpha>0.\ Find\ \alpha$ so that

$$i) P(X>1)=\frac{1}{3}, ii) P(|X|<1)=P(|X|>1)$$

36.Buses arrive at a specified stop at 15min intervals starting at 8A.M., that is they arrive at 8, 8.15,8.30 and soon. If the passenger arrives at the stop at a random time that is Uniformly distributed between 8 and 8.30A.M., find the probability that he waits (i) less than 5 min for a bus (ii)atleast 12 min for a bus.

37. Derive the MGF, mean and variance of Exponential distribution.

38.Establish the memory less property of Exponential distribution.

39. The mileage which car owners get with a certain kind of radial type is a random variable having an exponential distribution with mean 40.000km. Find the probabilities that one of these tires will last i)atleast 20.000km. ii) atmost 30,000km.

40.The time (in hours) required to repair a machine is exponential distribution parameter $\lambda=\frac{1}{2}$.

What is the i) probability that repair time exceed 2 hours. ii) conditional probability that a repair time takes atleast 10hours given that its duration exceeds 9 hours.

41.The daily consumption of milk in excess of 20,000 liters in a town is approximately exponentially distributed with parameter $\lambda=\frac{1}{3000}$. The town has daily stock of 35,000 liters. What is the probability that of 2 days selected at random the stock is insufficient for both days.

42.Derive the MGF, mean and variance of Gamma distribution.

43.Derive the mean and variance of Weibull distribution.

44. Suppose that the lifetime of a certain kind of emergency backup battery (in Hrs) is a random variable X having the weibull distribution $\alpha=0.1, \beta=0.5.$ Find the i) Mean lifetime of these batteries. ii)Probability that such a battery will last more than 300Hrs. iii) Probability that such a battery will not last 100Hrs

45. Each of the six tubes of a radio set has a life length (in years) which follows a weibull distribution parameters $\alpha = 25, \beta = 2.$ If these tubes function independently of one another. What is the probability that no tube will have to replaced during the first two months of operation.

46. If the life X (in year) of a certain type of car has a weibull distribution with the parameter $\beta = 2.$ Find the value of the parameter $\alpha,$ given that probability that the life of the car exceeds 5 years is $e^{-0.25}$. For these values of α and β find the mean and variance of X..

47. If the lifetime X in hrs of a component is modeled by a weibull distribution with $\beta = 2.$ Starting with a large number of components it is overed that 15 % of the components that have lasted 90hrs, fail before 100hrs. Find the parameter $\alpha,$

UNIT II

TWO DIMENSIONAL RANDOM VARIABLES

27. The joint pdf of X and Y is given by $f(x,y) = \frac{x+y}{21}, \; x = 1,2,3. \; y = 1,2.$ Find the marginal distributions.

28. If the joint pdf of (X,Y) is given by P(x,y)=k(2x+3y) , x=0,1,2. y=1,2,3. Find all the mariginal distridutios and the conditional distributions. Also find the probability distribution of (X+Y) and P(X+Y>3).

29. Let X and Y have the joint probability function

y\x	0	1	2
0	0.2	0.3	0.15
1	0.1	0.1	0.25

i)Find $P(X+Y>1)$. Ii) Are X and Y are independent random variables? Justify your answer.

30. The joint PDF of X and Y is given by $f(x,y) = \begin{cases} e^{-(x+y)}, & x \ge 0, \; y \ge 0 \\ 0, & otherwise \end{cases}$

Find i) the marginal density functions ii) are X and Y are independent. iii) P(X>Y) , P(X<1), P(X+Y<1) iv) $P(X<1 \cap Y<3)$

31. If the joint pdf X and Y is given by $f(x,y) = \begin{cases} \frac{1}{8}(6-x-y), & 0<x<2, \; 2<y<4. \\ 0, & otherwise \end{cases}$

Find i) $P(X<1 \cap Y<3)$ ii) P(X+Y<3) iii) P(X<1 / Y<3) iv) f (y/x=2) v)P(X<1,Y<3).

32. Given $f(x,y) = \begin{cases} cx(x-y), & 0<x<2, \; |y|<x \\ 0, & otherwise \end{cases}$. Find i)c ii)f(x), f(y). iii) Conditional density function of y for x.

33. The joint PDF of random variable X and Y is given by $f(x,y) = \begin{cases} cxy^2, & 0 \le x \le y \le 1 \\ 0, & otherwise \end{cases}$

i)Find the value of C. ii)Find the marginal density functions of X and Y. iii) Find the conditional density functions $f_{Y/X}\left(Y/X\right)$ and $f_{X/Y}\left(X/Y\right)$.

34. The joint PDF of random variable X and Y is given by $f(x,y) = k\,xy\,e^{-(x^2+y^2)}$, $x > 0,\ y > 0.$
Find the value of k. Prove that X and Y are independent.

35. The joint pdf of a two dimensional random variable (X,Y) is given by
$f(x,y) = xy^2 + \frac{x^2}{8},\ 0 \le x \le 2,\ 0 \le y \le 1.$ Compute $i) P(X > 1),\ P\left(Y < \frac{1}{2}\right),\ P(X < Y)$

$ii) P\left(X > 1 \big/ Y < \frac{1}{2}\right),\ P\left(Y < \frac{1}{2} \big/ X > 1\right)\quad iii) P(X + Y \le 1).$

36. The joint PDF of the random variables X and Y is given by $f(x,y) = \begin{cases} 8xy,\ 0 < x < 1,\ 0 < y < x \\ 0,\ otherwise \end{cases}$

Find i) $f_X(x),\ f_Y(Y)\ ii) f(y/x),\ f(x/y)\ iii) P(X < 1/2)$.

37. Given the joint PDF f(x,y) as $f(x,y) = \begin{cases} \frac{1}{4}(1+xy),\ |x| \le 1,\ |y| \le 1 \\ 0,\ otherwise \end{cases}$. Find the marginal and conditional PDF of X and Y. Show that X and Y are independent.

38. If the joint distribution function of X and Y is given by
$F(x,y) = \left(1-e^{-x}\right)\left(1-e^{-y}\right),\ for\ x > 0,\ y > 0$

$= 0,\ otherwise.$

Find i) the marginal density of X and Y. ii) Are X and Y are independent. iii) The conditional density function of X and y iv) P(X<1), P(1<x<3, 1<y<2) v) $P(X \le 1 \cap Y \le 1)\ (or)\ P(X \le 1,\ Y \le 1)$ vi) $P(X + Y \ge 1)$.

39. i)Calculate the correlation coefficient for the following heights(in inches)of fathers X and their sons Y.

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

ii)Find the coefficient of correlation between industrial production X and export Y from the following data.

Production X	55	56	58	59	60	60	62
Export Y	35	38	37	39	44	43	44

iii) Calculate the correlation coefficient for the following data

X	9	8	7	6	5	4	3	2	1
Y	15	16	14	13	11	12	10	8	9

40. Two random variables X and Y have the following joint PDF

$f(x,y) = \begin{cases} 2-x-y,\ 0 \le x \le 1,\ 0 \le y \le 1 \\ 0,\ otherwise \end{cases}$ Find var (X), Var (Y) Cov(X,Y) and ρ_{XY}.

41. The joint p.d.f of R.Vs X and Y is given by $f(x,y) = \begin{cases} 3(x+y),\ 0 \le x \le 1,\ 0 \le y \le 1,\ x+y \le 1 \\ 0,\ otherwise \end{cases}$

Find the i)Marginal PDF os X, ii) P(X+Y<1/2), iii) Cov(X,Y)

42. If $f(x,y)=\frac{1}{8}(6-x-y),\ 0\le x\le 2,\ 2\le y\le 4.$ Find the correlation coefficient between X and Y.

43. Determine the correlation coefficient between R.Vs X and Y whose j.p.d.f is

$$f(x,y)=\begin{cases} x+y,\ 0\le x\le 1,\ 0\le y\le 1 \\ 0,\ otherwise \end{cases}$$

44. From the following data find i) the two regression equations. ii) the coefficient of correlation between the marks in economics and statistics. iii) the mosy likely marks in statistics when marks in economics are 30.

Marks in Economics	25	28	35	32	31	36	29	38	34	32
Marks in Statistics	43	46	49	41	36	32	31	30	33	39

45. From the following data obtain two regression equations

X	6	2	10	4	8
Y	9	11	5	8	7

46. The two lines of regression are 8x-10y+66=0 and 40x-18y-214=0. The variance of X is 9. Find i)the mean values of X and Y. ii)the correlation coefficient between X and Y. iii) the standard deviation of Y.

47. If the joint PDF of a two dimensional random variable (X,Y) is given by $f(x,y)=x+y,\ 0\le x\le 1,\ 0\le y\le 1.$ Find the PDF of U=XY.

48. If X and Y each follow exponential distributions with parameter 1 and are independent. Find the PDF of U=X - Y.

49. The joint PDF of X and Y are given by $f(x,y)=e^{-(x+y)},\ x>0,\ y>0$. Find the PDF of $U=\frac{X+Y}{2}$.

50. Let X and Y be positive independent random variable with the identical probability density function

$f(x)=e^{-x},\ x>0.$ Find the joint probability density function of U=X+Y and V=X/Y. Are U and V independent?.

51. The lifetime of a certain kind of electric bulb may be considered as a random variable with mean 1200 hrs and standard deviation 250 hrs. Find the probability that the average lifetime of 60 bulbs exceeds 1250 hrs using central limit theorem.

52. i) A sample size 100 is taken from a population whose mean is 60 and variance is 400. Using CLT, with

probability can we assert that the mean of the sample will not differ from $\mu = 60$ by more than 4.

ii) A coin is tossed 10 times. What is the probability that of getting 3 or 4 or 5 heads. Use central limit theorem.

53. If $X_1, X_2, X_3, ..., X_n$ are Poisson Variates with parameter $\lambda = 2$,use the central limit theorem $p(120 \le S_n \le 160)$ where $S_n = X_1 + X_2 + ... + X_n$ and n=75.

54. A distribution with unknown mean μ has large variance equal to 1.5. Use central limit theorem , to find how large a sample should be taken from the distribution in order that the probability will be atleast 0.95 that the sample mean will be within 0.5 of the population mean. (2)

UNIT III
MARKOV PROCESSES AND MARKOV CHAINS

1. Explain the classification of random variables
2. Explain the classification of Markov process.
3. Obtain the steady state or long run probabilities for the population size of a birth death process. (or) Discuss the pure birth process and hence obtain its probabilities $P_{n(t),}$ $n = 1, 2, 3, ...$ mean and variance.

18. The probability distribution of the process $\{X(t)\}$ is given by

$$P(X(t) = n) = \begin{cases} \dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1, 2, 3, ... \\ \dfrac{(at)}{(1+at)}, & n = 0. \end{cases}$$

. Show that $\{X(t)\}$ is not stationary.

19. Show that the random process $X(t) = A\sin(\omega t + \theta)$ is wide sense stationary. If A and ω are constants and θ is uniformly distributed random variable in $(0, 2\pi)$.

20. Show that the random process $X(t) = A\cos(\omega t + \theta)$ where A and ω are constants and θ is uniformly distributed in $(-\pi, \pi)$. is wide sense stationary.

21. Two random process X(t) and Y(t) are defined by $X(t) = A\cos\omega t + B\sin\omega t$ *and* $Y(t) = B\cos\omega t - A\sin\omega t$. Show that X(t) and Y(t) are jointly wide sense stationary, if A and B are uncorrelated random variables with zero means and the same variances and ω is a constant.

22. Show that $X(t) = A\cos\lambda t + B\sin\lambda t$ (where A and B are two random variables) is wide sense stationary if i) E(A)= E(B)=0, ii) E(A^2)= E(B^2)=0 and E(AB)=0.

23. A fair die is tossed repeatedly. If X_n denotes the maximum of the numbers occurring in the first 'n' tosses , find the transition probability matrix P of the Markov Chain $\{X_{n, n\ge 0}\}$. Find also P^2 and $P\{X_2 = 6\}$.

24. The one step TPM of Markov chain $\{X_n\}$ n=1,2,3,… having three states 1,2,3 and

$$p^{(1)} = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$

and the initial distribution is $p^{(0)} = [0.7 \quad 0.2 \quad 0.1]$.

Find $i)P(X_2=3).\ ii)P\left(X_2=3/X_0=1\right)iii)P(X_3=2,X_2=3,X_1=3,X_0=2)$

25. Find the nature of the states of the Markov chain with the transition probability matrix $\begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix}$

26. The TPM of Markov chain with three states (0,1,2) is $p=\begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 2/4 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$ and the initial distribution of the chain is $P(X_0=i)=1/3$, $i=0,1,2.$.

Find $i)P(X_2=2).\ ii)P\left(X_2=1,X_0=0\right)iii)P(X_3=1,X_2=2,X_1=1,X_0=2)$ iv) Is the chain is irreducible? Explain

27. Consider the Markov chain $\{X_n, n\geq 0\}$ with state space $S=\{0,1\}$ and one step TPM $P=\begin{bmatrix} 1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ .i)Draw the state transition diagram. ii)Is the chain irreducible? Explain. Iii)Show that state 1 is transient.

28. Three boys A,B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the transition matrix and classify the states.

29. Find the mean and autocorrelation of Poisson process.

30. A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work iff a 6 appeared. Find i) the probability that he takes a train on the 3^{rd} day. ii) the probability that he drives to work in the long run.

31. Prove that the sum of two independent poisson process is again a poisson process. (or) State and prove the additive property of poisson process.

18. Prove that the difference of two independent poisson process is not a poisson process.

19. Prove that the poisson process is a Markov process.

20. Suppose that the customer arrive at a bank according to a poisson process with a mean rate of 3 per minute. Find the probability that during a time interval of (0,2) min i) Exactly 4 customers.ii) more than 4 customers arrive.

21. A gambler has Rs.2.He bets Re.1 at a time and wins Re.1 with probability ½ . He stops playing if he loses Rs.2 or wins Rs.4. (i) What is the tpm of the related Markov chain ? (ii) What is the probability that he has lost his money at the end of 5 plays?

UNIT IV
QUEUEING THEORY

1. Derive the formula for average number of customers in the system and queue..
2. Write short note on recurrent state, transient state.
3. Customers arrive at one-man barber shop according to a Poisson process with a mean inter arrival time of 12 minutes. Customers spend an average of 10 minutes in the barber's chair.

   i)What is the expected number of customers in the barber shop and in the queue?

ii)Calculate the % of time an arrival can walk straight in the barber's chair without having to wait.

iii)How much time can customer expect to spend in the barber's shop?

iv)Management will provide another chair and have another barber, when a customers waiting time in the shop exceeds 1.25hrs. How much must the average rate of arrivals increase to warrant a second barber?

v) What is the average time customers spend in the queue?

vi)What is the probability that the waiting time in the system is greater than 30 minutes?

vii)Calculate the % of customers who have to wait prior to getting into the barber's chair.

viii)What is the probability that more than three customers are in the system?

4. Customers arrive at the first class ticket counter of a theatre at a rate of 12 per hour. There is one clerk servicing the customers at the rate of 30 per hour.

   i)What is the probability that there is no customer at the counter?

   ii)What is the probability that there are more than 2 customers at the counter?

   iii)What is the probability that there is no customer waiting to be served?

   iv)What is the probability that a customer is being served and nobody is waiting?

   v)What is the probability that a customer has to wait for atmost 4 minutes in the queue.

5. Customers arriving at a watch repair shop according to poisson process at a rate of one per every 10 minutes and the service time is an exponential random variable with mean 8 minutes.

   i)Find the average number of customers in the shop.

   ii)Find the average time a customer spends in the shop.

   iii)Find the average number of customers in the queue.

   iv)What is the probability that the server is idle?

6. In a railway marshalling yard , goods trains arrive at a rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time distribution is also exponential with an average 36 min. Calculate the following

   i)the mean size of queue.

   ii)the probability that the queue size exceeds 10.

   iii)If the input of trains increase to an average of 33 per day. What will be the change in the above quantities?

7. Arrivals at a telephone booth are considered to be poisson with an average time of 12 min between one arrival and the next length of a phone call is assume to be distributed exponentially with mean 4 min.

   i)Find the average number of persons waiting in the system.

   ii)What is the probability that a person arriving at the booth will have to wait in the queue?

   iii) What is the probability that it will take him more than 10 min altogether to wait for the phone and complete his call?

   iv)Estimate the fraction of the day when the phone will be in use.

v)The telephone department will install a second booth, when convinced that an arrival has to wait on the average for atleast 3 min for phone. By how much the flow of arrivals should increase in order to justify a second booth?

vi)What is the average length of the queue, that forms from time to time.

8. Automatic car wash facility operates with only one bay. Cars arrive according to a Poisson process, with mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. If the service time for all cars is constant and equal ti 10 min. Determine L_s, L_q,W_s and W_q.

9. A supermarket has two girls attending to sales at the counters. If the service time for each customer is exponential with mean 4 min and if people arrive in a poisson fashion at the rate of 10 per hour.

   i)What is the probability of having to wait for service?

   ii)What is the expected % of idle time for each girl?

   iii)Find the average queue length and the average number of units in the system?

   iv)If a customer has to wait , what is the expected length of his waiting time?

10. There are 3 typist in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour.

    i)What fraction of the time all the typists will be busy?

    ii)What is the average number of letters waiting to be typed?

    iii) What is the average a letter has to spend for waiting and for being typed?

    iv)What is the probability that a letter will take longer than 20 min waiting to be typed and being typed?

11. A petrol pump station has 4 (or 2) pumps. The service time follows the exponential distribution with a mean of 6(or 4) min and cars arrive for service in a poisson process at the rate of 30(or10) cars per hour.

    i)What is the probability that an arrival would have to wait in line?

    ii)Find the average waiting time in the queue, average time spend in the system and the average number of cars in the system.

    iii) For what percentage of time would a pump be idle on an average?

12. A car service station has 2 bays where service can be offered simultaneously. Because os space limitations only 4 cars are accepted for servicing. The arrival process is Poisson with 12 cars per day. The service time in both the days is exponentially distributed with $\mu = 8$ car per day per bay. Find the average number of cars in the service station and the average time a car spends in the system

13. Patients arrive at a clinic according to poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate of 20 per hour.

    i)Find the effective arrival rate at the clinic.

    ii)What is the probability that an arriving patient will not wait?

    iii)What is the expected waiting time until a patient is discharged from the clinic?

14. For an M/M/2 queuing system with a waiting room of capacity 5, find the average number of customers in the system, assuming that arrival rate as 4 per hour and mean service time 30 minutes.
15. A one man barber has a total of 10 seats. Inter arrival times are exponentially distributed, and an average of 20 prospective customers arrive each hour at the shop. Those customers who find the shop full do not enter. The barber takes an average of 12 minutes to cut each customer's hair. Haircut times are exponentially distributed
    i)on the average how many haircut per hour will the barber complete?
    ii)on the average how much time will be spent in the shop by a customer who enters?
16. Explain an M/M/1 finite capacity queueing model and obtain an expressions for the steady state probabilities for the system size.
17. Find the average number of customers Ls in the M/M/1/N queueing system when $\lambda = \mu$.
18. A service station expects a customer every 4 minutes on the average. Service takes, on the average 3 minutes. Assume Poisson input and exponential service.
    i)What is the average number of customers waiting for service?
    ii)How long can a customer expect to wait for service?
    iii)What is the probability that a customer will spend less than 15 minutes waiting for and getting service?
    iv)What is the probability that a customer will spend longer that 10 minutes waiting for and getting service.

UNIT V
NON-MARKOVIAN QUEUES AND QUEUE NETWORKS

1. Derive the Pollaczek-Khintchine (P-K) formula for the M/G/1 queueing model.
2. Explain the model $(M/M/1):(G_D/\infty/\infty)$, derive the expression for L_q
3. Explain the model $(M/M/S):(\infty/FIFO)$, derive the average number of customers in the queue.
4. Define series queues, open queuing and closed queuing network..
5. Define Jackson network, open and closed Jackson network.
6. Automatic car wash facility operates with only one bay. Cars arrive according to a Poisson process, with mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. If the service time for all cars is constant and equal ti 10 min. Determine L_s, L_q,W_s and W_q if the service time
   i)is constant and equal to 10 min.
   ii)follows normal distribution with mean 12 min an S.D 3 min.
   iii)follows uniform distribution between 8 and 12 min.

iv)follows a discrete distribution with values 4,8 and 15 min with corresponding probabilities 0.2, 0.6 and 0.2.

7. In a big factory, there are a large number of operating machines and two sequential repair shops, which do the service of the damaged machines exponentially with respective rates of 1 per hour and 2 per hour. If the cumulative failure rate of all the machines in the factory is 0.5 per hour.

   i)Find the probability that both repair shops are idle.

   ii)Find the average number of machines in the service section of the factory.

   iii)The average repair time of a machine.

8. Consider asystem of two servers where customers from outside the system arrive at

   Server 1 at a possion rate and at a Poisson rate 4 and at server 2 at a poisson rate 5.The service rates for server 1 and 2 are 8 and 10 respectively.A customer upon completion of service at server 1 is likely to go to server 2 or leave the system; whereas a departure from server 2 will go 25 percent of the time to server 1 and will depart the system otherwise.Determine the limiting probabilities, L_s *and* L_q .